Competing by Restricting Choice: The Case of Search Platforms

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Abstract

We show that a two-sided platform can successfully compete by limiting the choice of potential matches it offers to its customers while charging higher prices than platforms with unrestricted choice. Starting from micro-foundations, we find that increasing the number of potential matches not only has a positive effect due to larger choice, but also a negative effect due to competition between agents on the same side. Agents with heterogeneous outside options resolve the trade-off between the two effects differently. For agents with a lower outside option, the competitive effect is stronger than the choice effect. Hence, these agents have higher willingness to pay for a platform restricting choice. Agents with a higher outside option prefer a platform offering unrestricted choice. Therefore, the two platforms may coexist without the market tipping. Our model may help explain why platforms with different business models coexist in markets using the stylized model of online dating.

Keywords: matching platform; indirect network effects; limits to network effects

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1 Introduction

Existence of network externalities in a market implies that utility of a customer from purchasing a product is in part determined by how many other customers buy the product (Ambrus and Argenziano, 2009). Seminal works analyzing network effects studied how addition of one more customer in a market influences another customer's utility, concluding that increasing the number of complements offered to a consumer may yield positive network effects (e.g., Katz and Shapiro (1985), Katz and Shapiro (1994)). Such positive externalities in turn will increase the attractiveness of the product, allowing firms to extract higher rents. If this is true, managers should provide customers with a large set of complements, as long as this can be achieved without sacrificing their quality.

Given this seemingly intuitive advice, it is surprising that there exist successful firms that operate by actively reducing the number of complements available to customers. These firms essentially limit the extent to which their customers benefit from indirect network effects. One may suspect that firms restricting choice should be at a competitive disadvantage. However, not only do these firms survive against competitors offering more complements, but they can also succeed in charging higher prices.

Consider, for example, (heterosexual) online dating industry, where customers draw value from larger numbers of opposite gender candidates and the companies spend substantial resources to attract people to their sites. Yet, firms in this market compete with very different business models. Most sites, such as Match.com, capitalize on their size by providing its members with unlimited access to thousands of other members in their database. Other sites, such as eHarmony.com, also pursue active member growth, but limit the number of new candidates that any member can see to seven, thereby actively reducing choice. At the same time, eHarmony charges a higher price than Match.[1] It is puzzling that despite access to fewer candidates, eHarmony's customers are willing to pay a premium of over 25%.[2]

Our explanation for this phenomenon is based on the interplay of two opposite effects that

[1] For details on eHarmony and Match see Piskorski et al. (2009).

[2] eHarmony claims to have a superior algorithm for matching people. Some point to this algorithm as the reason for customers' higher willingness to pay and hence for the price premium. Our model shows that even without a superior algorithm, eHarmony provides value to customers solely by limiting the choice.

arise when the number of candidates an agent meets on the platform increases.[3] On one hand, an agent is more likely to find an attractive match if the platform offers more candidates. On the other hand, he is less likely to be accepted by his chosen match if there are more candidates available to her. This is because the probability of being inferior to another candidate increases in the number of candidates on the same side. We call the former the *choice effect* and the latter the *competition effect*. Interestingly, the trade-off between the two effects is resolved differently for different individuals. For agents who have high disutility of staying alone or more immediacy for matching with a candidate, the competition effect is stronger than the choice effect. These agents will prefer on-line dating sites such as eHarmony, where both men and women see only a limited number of candidates. There, they can improve the probability of being accepted, even at the cost of seeing fewer candidates. In contrast, men who have higher value of staying alone (or less immediacy to find someone) do not find rejection as costly, and hence opt to join sites which offer a larger selection of candidates such as Match. The differences in the value of staying alone, which determine the sensitivity to the positive choice effect and the negative competition effect, can explain the coexistence of firms competing with different business models: those that offer more choice, and others that actively limit choice. Different offerings appeal to different types of customers. Contrary to expected, the site that offers fewer complements can thrive in a market aside of a site that offers more complements.

We formalize our insight in three steps. First, we build a stylized model of a two-sided dating market with men and women on the two sides, and derive properties of indirect network effects in this environment. Following seminal papers on network effects (e.g., Katz and Shapiro (1985)), we say that there is a positive network effect when the value of joining the platform increases in the number of agents participating in the platform on the same side.[4] Our model shows that the relative

[3]As Baldwin and Woodard (2009) point out, term 'platform' is used in three distinct but related fields: product development, technology strategy and industrial economics. We use it in the industrial economics sense. For examples of similar other platforms in that sense, we refer the reader to Hagiu (2014).

[4]This definition applies to both direct and indirect network effects. For the indirect network effect, in a two-sided platform, the benefit arises because having more agents on the same side increases the participation of agents on the other side. With more agents on the other side, the platform can offer a higher expected utility from a successful match to an agent. These properties play an important role in the platform's strategy via the choice effect and the competition effect, outlined above.

magnitudes of the two effects change as the number of candidates offered increases.[5] When only a few candidates are offered, there is little competition among same side agents and the choice effect dominates. This leads to a positive network effect. As the choice set increases, competition among same side agents increases and the competition effect eventually overwhelms the choice effect. The network effect then becomes negative. Although virtually all agents on a platform benefit from an increased choice set, gaining these benefits is *conditional* on the occurrence of a match. And the likelihood of matching decreases as the number of complements increases.

Second, we recognize that agents are heterogeneous in their need for immediacy of a match. Agents who have high costs of staying alone suffer more from the competition effect since competition increases the likelihood that these agents will remain single. Conversely, agents who have high utility from being alone are affected less by competition. Combining these two forces, our findings show that there are limits to the positive network effects. In particular, for each agent, there exists a threshold beyond which his utility decreases as more complements are offered. This gives the platforms an opportunity to extract rents by actively *reducing* the number of complements available.

Finally, we study how the limits to network effects influence strategies of platforms in a monopoly and duopoly and we characterize equilibria in such markets. Specifically, we show that a platform such as eHarmony, where both men and women see only a limited number of candidates, attracts agents who feel more immediacy to find a match, either because they have a lower utility from staying single, or less patience. Less patient agents join the platform with restricted choice because it increases their chance to find a match sooner, even at the cost of a possibly worse match due to meeting fewer number of candidates. Moreover, these agents are willing to pay a premium to participate in a platform restricting a choice set. Therefore the platform is active in the market and is profitable, even when it competes with a platform that offers more candidates and does not charge a fee to participate. More patient agents join the platform with more choice because meeting more candidates increases the expected value from the match, even if it reduces the expected probability of a successful match. This implies that a strategy of a platform in a competitive environment includes not only prices, but also the number of candidates it offers access to. It also explains why platforms such as eHarmony and Match can coexist in the market, as the

[5] We focus on environments where the number of candidates offered to both sides is the same.

mechanism we describe shows that the market does not need to tip in favor of the platform that offers more choice. Interestingly, eHarmony advertises itself as a website for people who are looking for a serious relationship, including marriage. This may be interpreted as targeting people with lower utility of being alone who want to get married relatively sooner.[6]

One may expect that when firms restrict choice, they do so by eliminating less compatible matches. Our explanation of the existence and popularity of restricted-choice platforms does not assume that platforms have any ability to recognize match potential, but assumes that the restriction happens by choosing candidates randomly. We also do not assume psychological aversion to abundant choices. It should be noted however that our model does not preclude these possibilities: if the platform that restricts choice also offers more compatible candidates and services, or if people have distaste for excessive choice, it may be even more successful than our model predicts.

The remainder of the paper is structured in following way: Section 2 provides a review of related literature. Section 3 sets up the model and then analyzes the strength of and limit to network effects, and how they depend on an agent's type. We show that as the number of candidates on both sides increases, positive network effects disappear for agents with lower utility of being alone. Section 4 investigates a market with a matching platform, and shows that there always exists an equilibrium where agents pay to participate in a platform that offers fewer candidates than the outside market, which is accessible for free. Section 4.2 extends the findings to the competition of two platforms, and Section 5 provides several extensions to the assumptions made in the main model. [[Finally,]]Section 6 discusses the importance of [[the key]] [[some of the]] assumptions in the model, and how relaxing these assumptions influences [[them may influence]] the results.

2 Related Literature

Our study studies network competition and network effects. Seminal works in this literature suggests that when platforms compete with each other, the platform offering the largest choice should

[6]Similarly, in the labor market, headhunters are used primarily by employers or candidates for whom the cost of not finding an acceptable match quickly is high (Khurana, 2004). In the real estate market, it is accepted that agents who opt for real estate brokers (as opposed to FSBO) are those who assign more value to quickly finalizing the transaction. In all these markets, people are willing to pay a premium for immediacy, which makes platforms that restrict choice more costly, as predicted by our model.

take over the market (e.g., Katz and Shapiro (1985)). Moreover, previous work on network effects often assumed that the presence of other agents on the platform exogenously increases utility, usually in a linear form (e.g., Rochet and Tirole (2003)). As a consequence, every additional agent on the platform increases payoff to others, no matter how many other agents are already available. We depart from such assumptions and derive the network effects from the micro-foundations in the defined matching environment. We find that aside from the positive (opposite-side) choice effect there is also a negative (same-side) competitive effect. And we study how the trade-off between the two effects allows for co-existence of platforms with different business models.

More recently, a number of papers examined the trade-off between the positive opposite-side effect and negative same-side effect to show how multiple firms can coexist in environments with network effects. These papers make different assumptions from ours, and so their results are determined by different forces. For example, Ellison and Fudenberg (2003) and Ellison and Mobius (2004) study competition between two auction sites. Similarly to our paper, they assume that agents are heterogeneous. In contrast, however, their agents choose auction sites (platforms) before they know their type, while ours are aware of their type prior to choosing their platform. Furthermore, they assume that the clearing price on every platform is determined by the ratio of buyers to sellers. Then, they show that multiple auction sites can coexist as long as they have the same buyer-to-seller ratio. Although this ratio is appropriate for auctions, it cannot be assumed to be the only crucial factor in other environments. Thus, our model builds micro-foundations of the trade-off between choice and competition to show that the number of candidates offered on a platform explains why multiple platforms can coexist in matching environments. Several other economists have identified the complications that rise from increasing the number of options in a network. Calvo-Armengol and Zenou (2005) for example, suggested in the context of a labor market that being connected to too many others in a random matching network can result in frictions in the network and reduce the probability of a match in a job network. This study, however, does not identify the limits of positive network effects for each agent but arbitrarily assumes the same limit for all.

Our model is perhaps closest to Damiano and Li (2007), who examine why a revenue-maximizing monopolist would establish many platforms with different entry prices to separate and match different types of agents. Their model assumes that agents are heterogeneous in productivity and have different reservation utilities. They find that platforms can charge different prices to separate high productivity agents and allow them to match with each other. This is similar to our result whereby price separates agents with low utility of being unmatched from others. However, they assume that on every platform established by the monopolist agents can only consider one candidate. We relax this assumption to show that platforms that reduce the number of candidates are valuable to agents with low utility of staying alone. In the discussion section we examine the relationship between our paper and Damiano and Li (2007) in more detail. [[Make sure we still do!!]]

An emerging literature in strategy explores competitive interaction between organizations with different business models. Casadesus-Masanell and Ghemawat (2006) and Economides and Katsamakas (2006), for example, study duopoly models in which a profit-maximizing competitor interacts with an open-source competitor. Casadesus-Masanell and Hervas-Drane (2010) study competitive interaction between a high-quality incumbent that faces a low-quality ad-sponsored competitor. Finally, Casadesus-Masanell and Zhu (2010) analyze competitive interactions between a free peer-to-peer file-sharing network and a profit-maximizing firm that sells the same content at positive price, and distributes digital files through an efficient client-server architecture. In our paper, firms could be seen as competing with different business models, as one matching platform deliberately limits the choice while competing against one that offers unlimited choice within its data base. We study forces in the market that allow such competition to be successful.

Our study also relates to the literature on 'co-opetition' (e.g. Brandenburger and Nalebuff (1996); MacDonald and Ryall (2004)). In their widely-cited book, Brandenburger and Nalebuff (1996) argue that seemingly competing product offerings may in fact act like complements and yield positive network effects. In many ways, the two forces we describe in our study, namely the 'competition' and 'choice' effects point out to the complementary and substitutionary characteristics of candidates on a platform. Each candidate simultaneously acts as a complement for agents on the opposite side and a substitute for agents on the same side of the market, resonating with the arguments made by Brandenburger and Nalebuff (1996) and MacDonald and Ryall (2004). By deriving network effects in our model from microfoundations, we show how the interplay between the competitive and cooperative forces affects the properties of the network effect. Moreover, we study how those forces affect successful strategies of competing firms.

We show why some agents may prefer an environment with less choice. The reasons why rational agents would make such decisions might be of relevance to the branch of behavioral economics and psychology dealing with the negative outcomes of increasing choice. The studies in this area suggest that providing higher number of choices might eventually decrease the satisfaction and happiness levels of consumers, suggesting behavioral mechanisms such as decision fatigue, choice overload, and cognitive costs (e.g., Iyengar et al. (2006); Schwartz and Ward (2004)). Our study shows that even in the absence of behavioral considerations, there is an economic explanation for why consumers may not always be happier and more satisfied with higher number of choices.

3 Matching Environment

We use a stylized example of a two-sided heterosexual dating market for stability of reference, and call the two sides 'men' (denoted by letter m) and 'women' (denoted by letter w). On each side of the market, there is a continuum of agents of measure 1. Agents are heterogeneous with respect to how much utility they receive from being alone, denoted by variable $\mathfrak{a}$ with c.d.f. $G(\mathfrak{a})$ on the interval $[0, 1]$. The value $\mathfrak{a}$ is private information for each agent.

There are two stages in the 'matching' game. In the first stage, every agent meets some fixed number of N agents from the other side of the market.[7] The number of candidates, N, is the comparative statics parameter in this model: we investigate how an increase in this parameter influences the expected payoff of agents. In the second stage, all agents simultaneously make at most one offer.[8] A match between man m and woman w happens if m made his offer to w and w has made her offer to m (i.e., the offer has been "reciprocated" or "accepted").

Let $\Lambda^m(w)$ represent how much the man m likes being with the woman w, and $\Lambda^w(m)$ represent how much the woman w likes being with the man m. We assume that both the woman's and the man's liking functions are drawn from some generalized distribution with the c.d.f. $G(\Lambda)$ on the

[7] We consider markets where the two sides are treated symmetrically. Platforms literature has shown the potential of asymmetric treatment of the two-sides (e.g., Parker and van Alstyne, 2005). However, in many markets firms are restricted to treat both sides symmetrically, for legal or technical reasons.

[8] The assumption that limits agents to only one offer is meant to reflect the fact that people are able to pursue only limited number of possible relationships. This restriction applies also to other matching markets. In labor market, for example, although the employers screen dozens of applicants, they may have capacity for a much smaller number of interviews. Because this is potentially restrictive assumption, Section 5.1 considers other offer-making procedures and shows that the results of this restrictive assumption hold also under more realistic procedures. The one-offer assumption made throughout the paper simplifies the analysis and the intuition behind the results.

interval $[0,1]$.[9] When a man m meets a woman w,[10] he learns $\Lambda^m(w) \in [0,1]$, i.e., how much he will like being in a relationship with her. Similarly every woman w learns $\Lambda^w(m) \in [0,1]$ about every man m she meets.

For a man m_i with $\mathfrak{a}^{m_i}$ to make an offer to a woman w_i, two conditions must be satisfied. First, he must like woman w_i more than staying alone ($\Lambda^{m_i}(w_i) > \mathfrak{a}^{m_i}$). Second, he must like w_i more than the other $N-1$ women he meets ($\Lambda^{m_i}(w_i) > \Lambda^m(w_j),\ \forall j = 1, 2, ...N, j \neq i$). For a successful match, the same must hold for the woman w_i; she must like m_i more than she likes being alone ($\Lambda^{w_i}(m_i) > \mathfrak{a}^{w_i}$), and more than the other $N-1$ men she meets ($\Lambda^{w_i}(m_i) > \Lambda^{w_i}(m_j),\ \forall j = 1, 2, ...N, j \neq i$). When all of these conditions are satisfied, offers of m and w are reciprocated and a successful match takes place. If their offers are reciprocated, agents receive their respective payoffs of $\Lambda^{m_i}(w_i)$ and $\Lambda^{w_i}(m_i)$. If an offer was not reciprocated (i.e., it is "rejected") the agent who made the offer remains unmatched, and he receives his or her $\mathfrak{a}$. The game ends with these payoffs.

An important assumption in our framework is the 'independence' assumption: Λ is independent of other values of Λ and $\mathfrak{a}$. The function Λ is subjective in our model: the utility of w from being matched to m, $\Lambda^w(m)$, is intrinsic to w and is privately known by her. It does not depend on $\mathfrak{a}^m$. In other words, our model assumes that agents' dating preferences are 'horizontally', rather than 'vertically' differentiated. This assumption has also other consequences. First, how much two agents like each other is not correlated. This implies that the extent to which a man likes a particular woman is independent of how much she likes him. Next, how much a man (woman) likes a woman (man) is independent of how much the other men (women) like her (him). Finally, an agent can like two different agents at different rates. That is, how much m likes w_1 is also independent of how much he likes another woman w_2.

Existing literature mostly focuses on agents' attributes that are similarly desired by all potential partners. Such attributes can be characterized as objective "quality" (e.g., Damiano and Li, 2007, 2008; McAfee, 2002, Becker (1973)). We want to study markets where preferences are more subjective—how much I like a potential romantic partner is different from other agent's liking; similarly, how much I like a certain house is subjective. In the main model of the paper we take an

[9] Where there is no risk of confusion, the notation is simplified by dropping superscripts. For example, $\Lambda^m(w)$ may be simplified to Λ.

[10] If m meets w, then it must be that w meets m.

extreme view and assume full subjectivity. This can be justified when considering candidates within a certain category (e.g., the same education, status, sense of humor). For a broader approach, it is more appropriate to allow for partial correlation between agents' preferences. Section **??** explores a matching environment where Λ's are correlated with $\mathfrak{a}$'s.

Lemma 1 identifies important characteristics of the described matching market.

Lemma 1 *In a market with N candidates:*

(i) For every agent the probability of being rejected by a candidate on the other side of the market is

$$Pr(rej|N) = \frac{N}{N+1}.$$

That is, the probability of being accepted is $\frac{1}{N+1}$.

(ii) An agent $\mathfrak{a}$ matches successfully with probability

$$(1 - Pr(rej|N))\left(1 - G^N(\mathfrak{a})\right).$$

(iii) For an agent $\mathfrak{a}$ the expected value of a match, conditional on being accepted, is

$$N \int_{\mathfrak{a}}^{1} G^{N-1}(\Lambda) g(\Lambda) \Lambda d\Lambda.$$

(iv) The total expected payoff for agent $\mathfrak{a}$ is

$$EU(\mathfrak{a}|N) = \underbrace{\left[1 - (1 - Pr(rej|N))\left(1 - G^N(\mathfrak{a})\right)\right]}_{prop\ of\ \textbf{not}\ matching} \mathfrak{a} + \underbrace{(1 - Pr(rej|N))}_{prob\ of\ acceptance} \cdot \underbrace{N \int_{\mathfrak{a}}^{1} G^{N-1}(\Lambda) g(\Lambda) \Lambda d\Lambda}_{exp\ payoff\ if\ accepted}$$

Proof: See Appendix, page 31.

When a man makes an offer to a woman, he does not know her $\mathfrak{a}$ or how much she likes him versus the other men she has met. A priori, she is equally likely to make an offer to any of the men, or not make an offer at all. Therefore, the probability that the offer is reciprocated by the woman is $\frac{1}{N+1}$. This is equivalent to the probability of being rejected $\frac{N}{N+1}$. This result is captured in part (i) of Lemma 1.

Whether an agent matches successfully depends on two factors: whether the agent wants to make an offer, and whether the offer is reciprocated. From part (i) of Lemma 1, we know that the offer is reciprocated with probability $\frac{1}{N+1}$. Whether the agent wants to make an offer depends on his $\mathfrak{a}$. Because the expected probability of rejection is the same for all candidates, the optimal strategy is to make an offer to the best candidate, if that candidate is above agent's $\mathfrak{a}$. With probability $G^N(\mathfrak{a})$ all N candidates are below $\mathfrak{a}$. Thus, with probability $1 - G^N(\mathfrak{a})$ the best candidate's Λ is above $\mathfrak{a}$. The probability of matching successfully is captured in part (ii) of Lemma 1.

If the offer is accepted, it means that the agent has matched with the highest Λ among the N candidates, and this highest Λ was above $\mathfrak{a}$. Thus, the agent receives the payoff equal to the expected value of the maximal Λ given that it is above $\mathfrak{a}$. This is formalized in part (iii) of Lemma 1.

Part (iv) of Lemma 1 puts all the parts together and formalizes the expected payoff of an agent $\mathfrak{a}$ in a market with N candidates. We say that positive *network effects* are present if increasing the size of accessible network, N, increases agent's expected payoff. When increasing the size of accessible network decreases agent's expected payoff, the network effects are negative.

A number of properties follow directly from Lemma 1. Two of them, choice effect and competition effect, characterized below, play especially important role in our analysis.

Corollary 2 (Choice Effect) *For any $\mathfrak{a} < 1$, expected value of a match, conditional on successfully matching, is nondecreasing with the number of candidates (N):*

$$\frac{\partial \left(N \int_{\mathfrak{a}}^{1} G^{N-1}(\Lambda) g(\Lambda) \Lambda d\Lambda \right)}{\partial N} \geq 0.$$

Proof: See Appendix, page 31.

Corollary 2 states that the expected value from a successful match increases when an agent meets more candidates. As N increases, conditionally on a successful match, the agent can expect to match with a woman of his higher liking. We refer to this effect as the *choice effect*. This would suggest that an agent can achieve higher expected utility when dating in a market with more candidates. And in many environments this effect is the driver of the positive network effects. However, we also need to take into account the *competition effect*, as stated in Corollary 3. The probability that agent $\mathfrak{a}$'s offer will be accepted is decreasing with N. With more candidates, each woman has more men to choose from, i.e., every man has more competition. This decreases the probability that a woman w wants to match with man m, when m wants to match with w.

Corollary 3 (Competition Effect) *For every agent* $\mathfrak{a} < 1$*, the probability of being rejected is increasing in* N*.*

Proof: Follows directly from part (i) of Lemma 1.

So, does the market offering larger number of candidates (a larger "dating network") make the agents better off? Corollaries 2 and 3 document effects going in opposite directions. The expected pay-off of an agent from joining the platform depends on both the choice and the competition effect. If the expected payoff for an agent increases as N increases, there is a positive indirect network effect: having more agents on the same side increases the agent's utility, because it is combined with increasing the number of candidates on the other side of the market. Proposition 4 shows that there are positive network effects, but they disappear as N and $\mathfrak{a}$ increase.

Proposition 4 (Limits to Positive Network Effects)
(i) For every $\mathfrak{a}$*, there exists* $\bar{N}(\mathfrak{a})$ *such that* $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N)$ *is positive for* $N < \bar{N}(\mathfrak{a})$*, and negative for* $N \geq \bar{N}(\mathfrak{a})$*.*
(ii) $\bar{N}(\mathfrak{a})$ *is non-decreasing in* $\mathfrak{a}$*.*

Proof: See Appendix, page 32.

Proposition 4 coins the first main insight of this paper: for every agent, there exists a limit beyond which there are no positive network effects. Figure 1 illustrates how this limit to the network effect varies with $\mathfrak{a}$. The choice effect, stated in Corollary 2, declines in strength as N increases. Each additional candidate increases the expected value of a successful match by a smaller amount than the previous one. At the same time, the competition effect, stated in Corollary 3, increases in N. The positive network effect experienced by agent $\mathfrak{a}$ declines in strength as N increases, until it reaches its limit at $\bar{N}(\mathfrak{a})$. Above that level, an increase of N decreases agent's expected payoff: above $\bar{N}(\mathfrak{a})$ the network effect is negative. Additionally, part (ii) of Proposition 4 states that for agents with higher $\mathfrak{a}$'s, $\bar{N}(\mathfrak{a})$ is larger. For agents with low $\mathfrak{a}$ it is already likely

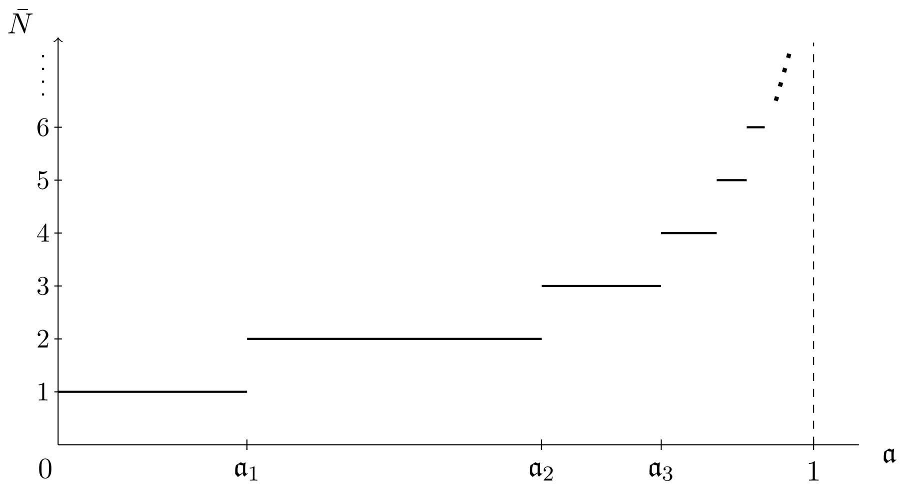

Figure 1: Limit to the network effect as a function of $\mathfrak{a}$.

that few candidates provides matching value above $\mathfrak{a}$. Meeting more candidates does not increase this probability enough to offset the increased probability of having the offer rejected. However, for agents with high $\mathfrak{a}$, the increase in the probability that at least one candidate is better than $\mathfrak{a}$ offsets the increased probability of having an offer rejected, as the agent meets an additional candidate. If the number of candidates N is large enough, agents with low $\mathfrak{a}$ prefer that N was lower, while agents with high $\mathfrak{a}$ prefer that N was even higher. That is, agents with low $\mathfrak{a}$ feel that they are in a market with "too many candidates." This property is mainly driven by the assumption that agents can court a limited number of candidates.[11] The larger the pool, the smaller the probability that the agent is within the limited number of courted candidates.

Our analysis so far implies that network effects (both the strength and direction) depend not only on the agent's type, $\mathfrak{a}$, but also on how much competition the network supports, N. In contrast, most of the literature on platform competition assumes that the number of agents on the other side of the market enter the payoff function linearly, i.e., every agent on the other side of the network contributes the same amount of expected payoff (e.g., Rochet and Tirole, 2003). If the competitive effect is included, every additional agent on the same side of the market also affects the payoff in the same way. The positive network effect is present when choice outweighs

[11] Our main model assumes that an agent can court only one candidate. In Section 5.1 we show that the results hold when every agent can court an arbitrary but fixed number of candidates.

the competition effect. But since the effects are assumed to be constant, the positive network effect never weakens or disappears, and it is always better for agents when the accessible network increases (more choice and more competition). In such a set-up it is always profit maximizing for a platform to offer access to all the agents who joined.[12] Moreover, it would not be possible for a platform that restricts choice to attract agents away from a market with more choice and more competition.

In the next section, we show that given the described properties of the matching market and limits to network effects, a platform can successfully operate in a market by offering less choice compared to the other existing options in the market.

4 Matching Platforms

4.1 Matchmaking Platform and Non-strategic Outside Market

The trade-off outlined in the previous section demonstrates that some agents prefer a market with fewer candidates. We have shown that the trade-off varies with the type of agent, $\mathfrak{a}$. In this section, we explore strategic opportunities that those properties offer to a matching platform. In particular, we focus on the fact that a platform may earn positive profits when providing *fewer* candidates than the outside market to its participants, as long as it offers fewer candidates to agents on both sides.

Let the *outside market* be a decentralized, non-strategic market, where agents meet Ω candidates and pay no fee. There is also a matching platform offering $N < \Omega$ candidates, and charging a positive participation fee f. Agents decide whether to participate in the platform or stay in the outside market.

As we shall see, providing fewer candidates restricts the choice, but also restricts the competition, which results in lower rejection probability. Only agents with low enough $\mathfrak{a}$ prefer to participate in the platform at a given positive fee. Agents with higher $\mathfrak{a}$ prefer to stay in the outside market. In other words, agents for whom the competition effect is large compared to the choice effect are willing to pay a positive fee to participate in such a platform. Therefore, candidates that

[12]Notice that in our environment the platform would have many more agents joining but offer access to N candidates.

can be met in the platform have $\mathfrak{a}$'s drawn from a truncated distribution. This property further influences the rejection probabilities in the platform and in the outside market.

To characterize equilibria in such a market, first notice that an environment in which *all* agents stay in the outside market is always an equilibrium. However, there also always exist equilibria where some agents participate in the platform. We focus our investigation on the equilibria where the platform is active (i.e., some agents participate in the platform). Especially, we show that for $N < \Omega$, there always exists an equilibrium where some agents pay a positive price to participate in the platform. To characterize this equilibrium, we start by considering how an agent's willingness to pay for joining the platform changes with his or her type, $\mathfrak{a}$. An agent is willing to pay up to the additional utiilty that the platform provides above the outside market, i.e., $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N) - EU(\mathfrak{a}|\Omega)$.When an agent makes his individual decision about whether to join the platform, he takes others' actions as given, and thus platform's fee and rejection probabilities are constant from the point of view of the agent. (In an equilibrium, the rejection probabilities are determined by agents' participation decisions.) Whereas the rejection probabilities and the fee are the same for all agents, the expected payoff depends on $\mathfrak{a}$.

Lemma 5 *For any given $Pr(rej|N)$ and $Pr(rej|\Omega) > Pr(rej|N)$ and for any Ω and $N < \Omega$, the willingness to pay $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N, Pr(rej|N)) - EU(\mathfrak{a}|\Omega, Pr(rej|\Omega))$ is positive and decreasing for $\mathfrak{a} \in [0, \tilde{\mathfrak{a}})$, where $0 < \tilde{\mathfrak{a}} \leq 1$.*

Proof. See Appendix, page 34.

An agent prefers to join the platform only if the benefit of joining outweighs the fee, i.e., $WTP(\mathfrak{a}) > f$. Therefore, it follows from Lemma 5 that for any $Pr(rej|N)$ and $Pr(rej|\Omega) > Pr(rej|N)$ and some positive fee f,[13] there exists $\mathfrak{a}^* \in (0, 1)$ such that agents with $\mathfrak{a} \in [0, \mathfrak{a}^*)$ strictly prefer to join the platform at f, agents with $\mathfrak{a} \in (\mathfrak{a}^*, 1]$ prefer to stay outside, and agents $\mathfrak{a}^*$ are indifferent.

Under this circumstance, the probability of rejection for an agent is affected by the fact that $\mathfrak{a}$ of candidates is not drawn from the whole distribution, but from a subinterval $[\underline{\mathfrak{a}}, \overline{\mathfrak{a}}] \subset [0, 1]$. Then,

[13] As long as the fee is not prohibitively high, i.e., $f < EU(\mathfrak{a} = 0|N, Pr(rej|N)) - EU(\mathfrak{a} = 0|\Omega, Pr(rej|\Omega))$.

the probability of rejection is

$$Pr\left(rej|X, \mathfrak{a} \in [\underline{\mathfrak{a}}, \overline{\mathfrak{a}}]\right) = 1 - \frac{1}{X} + \frac{1}{X(X+1)} \cdot \frac{G^{X+1}(\overline{\mathfrak{a}}) - G^{X+1}(\underline{\mathfrak{a}})}{G(\overline{\mathfrak{a}}) - G(\underline{\mathfrak{a}})}.$$

Notice that $Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}^*)) < Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}^*, 1])$ for any Ω, $N < \Omega$ and $\mathfrak{a}^*$. This is because $N < \Omega$, and because lower $\mathfrak{a}$'s join the platform. With $Pr(rej|N) < Pr(rej|\Omega)$, the premise of Lemma 5 is satisfied. It is also worth noting that when the rejection probability is higher in the platform than in the outside market, i.e., $Pr(rej|N) > Pr(rej|\Omega)$, no agent joins the platform at any positive f.[14] Therefore, there does not exist an equilibrium with active platform and $Pr(rej|N) > Pr(rej|\Omega)$.

The platform sets its fee, f, with the objective of maximizing its profit.[15] In equilibrium it must be that $EU(\mathfrak{a}^*|N, Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}^*))) - EU(\mathfrak{a}^*|\Omega, Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}^*, 1])) = f$. This condition characterizes the threshold $\mathfrak{a}^*$ on which the market settles for any f chosen by the platform. Moreover, $\mathfrak{a}^*$ uniquely characterizes the rejection probabilities, for given N and Ω. Therefore, we can solve the problem as if the platform was choosing $\mathfrak{a}^*$ directly instead of choosing f.

Platform's profit is $\mathfrak{a}^* \cdot f(\mathfrak{a}^*)$. Unsurprisingly, for higher fees fewer agents find it worthwhile to participate in the platform, and more agents join at lower fee. Nobody joins (i.e., $\mathfrak{a}^* = 0$) when f rises to 1. And to capture the whole market, the platform needs to set $f = 0$.[16] However, for intermediate fees (i.e., $\mathfrak{a}^* \in (0, 1)$) the profit is positive. Therefore, for any Ω and $N < \Omega$, there exists an equilibrium with an active platform.

Proposition 6 *Suppose that in the outside market agents meet Ω candidates, and that there is a platform offering $N < \Omega$ candidates. For any Ω and $N < \Omega$, there exists an equilibrium where the platform maximizes profit by charging a positive fee f. In this equilibrium there exists a threshold $\mathfrak{a}^* \in (0, 1)$ such that agents with $\mathfrak{a} \in [0, \mathfrak{a}^*)$ join the platform, agents with $\mathfrak{a} \in (\mathfrak{a}^*, 1]$ stay in the outside market, and agents with $\mathfrak{a} = \mathfrak{a}^*$ are indifferent.*

Proof. See Appendix, page 36.

[14] This can be shown by the same reasoning as in the proof of Lemma 5. See Corollary A.3 in Appendix, page 35.

[15] We assume that all the costs for the platform are fixed costs, and the marginal cost is 0. Thus, the profit maximization is equivalent to revenue maximization.

[16] See Corollary A.4 in Appendix, page 35.

Proposition 6 shows that there is a profitable strategy of limiting the number of candidates. When the platform provides fewer candidates than the outside market, a non-empty interval of agents joins the platform at a positive fee. Interestingly, the platform does not find it profitable to serve the whole market. The rejection probability in the platform is lower than the rejection probability in the outside market for two reasons. First, agents face less competition in the platform, due to $N < \Omega$. Second, agents in the platform are more likely to make and accept an offer, since they have lower utility of being alone. The outside market offers more candidates. Larger number of candidates increases the expected value of a match if matching is successful. For agents with higher utility of being alone, the positive choice effect outweighs the negative competition effect. Those agents prefer the outside market, which offers more choice and more competition. Agents with lower utility of being alone, however, prefer to join the platform, where they have less competition, but also less choice.

4.2 Competing Platforms

The previous section analyzed the optimal strategy of a matching platform facing a non-strategic outside market. In this section we investigate equilibrium in a market where there are two platforms setting their access fees to maximize their profits. We show that a platform offering fewer candidates can profitably coexist in the market with a platform offering a larger number of candidates. Moreover, the platform with fewer candidates charges higher price.

Suppose that one platform offers M_1 candidates, and the other $M_2 > M_1$. We use M_i to denote both the platform and the number of candidates it offers. Each platform $i = 1, 2$ charges f_i to maximize its profit. We maintain the assumption of single-homing. An agent who does not join either of the platforms remains unmatched.[17]

Consider an agent making a decision about which platform to join, if any, given the decisions of everyone else. That is, from the point of view of an individual agent the fees charged by the platforms, and the respective rejection probabilities are constant. (In equilibrium, however, they are determined by participation decisions.)

[17] We make this assumption because the point of this section is to show the interaction between the two platforms, and the assumption allows for mathematical simplicity of the proofs.

Lemma 7 *For any given* $Pr(rej|M_2)$*,* $EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a}$ *is positive and decreasing in* $\mathfrak{a}$*. Moreover, for* $\mathfrak{a} = 1$*,* $EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a} = 0$*.*

Proof. See Appendix, page 37.

For a given probability of rejection, consider a positive f_2.[18] By Lemma 7 there exists $\mathfrak{a}_2^* \in (0, 1)$ such that agent $\mathfrak{a}_2^*$ is indifferent between joining platform M_2 at f_2 and staying unmatched, i.e., $EU(\mathfrak{a}_2^*|M_2, Pr(rej|M_2)) - f_2 = \mathfrak{a}_2^*$. All agents $\mathfrak{a} > \mathfrak{a}_2^*$ strictly prefer staying unmatched to joining M_2, while agents $\mathfrak{a} > \mathfrak{a}_2^*$ strictly prefer joining M_2 to staying unmatched.

Applying Lemma 5 to $M_1 < M_2$ and any $Pr(rej|M_1)$ and $Pr(rej|M_2)$ such that $Pr(rej|M_1) < Pr(rej|M_2)$ yields Corollary 8.

Corollary 8 *For any given* $Pr(rej|M_1)$ *and* $Pr(rej|M_2)$ *such that* $Pr(rej|M_1) < Pr(rej|M_2)$*,* $EU(\mathfrak{a}|M_1, Pr(rej|M_1)) - E(\mathfrak{a}|M_2, Pr(rej|M_2))$ *is positive and decreasing for* $\mathfrak{a} \in [0, \bar{\mathfrak{a}})$*, where* $0 < \bar{\mathfrak{a}} \leq 1$*. Moreover,* $EU(\bar{\mathfrak{a}}|M_1, Pr(rej|M_1)) - E(\bar{\mathfrak{a}}|M_2, Pr(rej|M_2)) = 0$

Consider now any $f_1 > f_2$.[19] By Corollary 8 there exists $\mathfrak{a}_1^* \in (0, 1)$ such that agent $\mathfrak{a}_1^*$ is indifferent between joining platform M_1 at f_1 and joining M_2 at f_2, i.e., $EU(\mathfrak{a}_1^*|M_1, Pr(rej, M_1)) - f_1 = EU(\mathfrak{a}_1^*|M_2, Pr(rej, M_2)) - f_2$. All agents $\mathfrak{a} > \mathfrak{a}_1^*$ strictly prefer M_2 to M_1, and all agents $\mathfrak{a} < \mathfrak{a}_2^*$ strictly prefer M_1 to M_2.

If $\mathfrak{a}_1^* > \mathfrak{a}_2^*$, then no agent chooses to join platform M_2. This is because agents with $\mathfrak{a} < \mathfrak{a}_2^*$ prefer M_1 to M_2; agents with $\mathfrak{a} > \mathfrak{a}_1^*$ prefer staying unmatched rather than to join M_2; and for agents with $\mathfrak{a}_2^* < \mathfrak{a} < \mathfrak{a}_1^*$ both staying unmatched and joining platform M_1 are more attractive than M_2.

When $\mathfrak{a}_1^* < \mathfrak{a}_2^*$, then agents with $\mathfrak{a} < \mathfrak{a}_1^*$ choose M_1 (they prefer M_2 to staying unmatched, but M_1 to M_2); agents with $\mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*)$ choose M_2 (they prefer it both to M_1 and to staying unmatched); and agents with $\mathfrak{a} > \mathfrak{a}_2^*$ choose staying unmatched to either of the platforms. Notice also, that in such a case then the resulting rejection probabilities are indeed $Pr(rej|M_1, \mathfrak{a} \in [0, \mathfrak{a}_1^*)) < Pr(rej|M_2, \mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*))$.

Given the decisions of the agents, platforms decide on their strategies, i.e., setting the fees. Notice, however, that f_1 and f_2 uniquely characterize $\mathfrak{a}_1^*(f_1, f_2)$ and $\mathfrak{a}_2^*(f_1, f_2)$; moreover, $\mathfrak{a}_1^*$ and

[18] As long as the fee is not prohibitively high, i.e. $f_2 < EU(\mathfrak{a} = 0|M_2, Pr(rej|M_2)$.

[19] As long as the fee is not prohibitively high, i.e. $f_1 < EU(\mathfrak{a} = 0|M_1, Pr(rej|M_1) - EU(\mathfrak{a} = 0|M_2, Pr(rej|M_2) + f_2$.

$\mathfrak{a}_2^*$ uniquely characterize $Pr(rej|M_1, \mathfrak{a} \in [0, \mathfrak{a}_1^*))$ and $Pr(rej|M_1, \mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*))$. Therefore, we can think of the platforms as choosing a_i^* given a_j^*, instead of f_i given f_j.

Platforms' profits are a product of their fees and the measure of agents who join them. First, notice that platform M_1 would never set $\mathfrak{a}_1^* = 1$, as it would require $f_1 = 0$, and would result in 0 profits, while positive profits for other $\mathfrak{a}_1^*$'s are available. Similarly, M_1 never sets $\mathfrak{a}_1^* = 0$, as it also results in 0 profits.

Next, notice that platform M_2 would never set $\mathfrak{a}_2^* \leq \mathfrak{a}_1^*$, as it would bring it 0 profit. Also, setting $\mathfrak{a}_2^* = 1$ would require $f_2 = 0$, and would result in 0 profits. Thus, in an equilibrium $0 < \mathfrak{a}_1^* < \mathfrak{a}_2^* < 1$. In the proof of Proposition 9 we show that such an equilibrium exists.[20]

Proposition 9 *Suppose that in the market there are two matching platforms which offer M_1 and $M_2 > M_1$ candidates, respectively. For any M_1 and $M_2 > M_1$ there exists an equilibrium where platforms charge positive fees f_1 and $f_2 < f_1$, respectively, and there are two thresholds $\mathfrak{a}_1^*$ and $\mathfrak{a}_2^*$ such that $0 < \mathfrak{a}_1^* < \mathfrak{a}_2^* < 1$, and agents with $\mathfrak{a} \in [0, \mathfrak{a}_1^*)$ participate in platform M_1, agents with $\mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*)$ participate in platform M_2, agents with $\mathfrak{a} \in (\mathfrak{a}_2^*, 1]$ remain unmatched. Agents with $\mathfrak{a} = \mathfrak{a}_1^*$ are indifferent between M_1 and M_2, and agents with $\mathfrak{a} = \mathfrak{a}_2^*$ are indifferent between platform M_2 and remaining unmatched.*

Proof. Please see Appendix page 37 for the proof.

Proposition 9 establishes that two strategic matching platforms can profitably coexist in the market. By the same logic as in the proposition, we can see that more such platforms—each offering a different number of candidates—could profitably coexist in the market. As the number of matching platforms operating in the market increases, each attracts a smaller interval of agents, thus earning smaller profits. Positive fixed cost of operation, or entry cost, may hold more firms from entering the market, as they would not be able to cover those costs. Without fixed cost, and with continuum of agents, there could be an infinite number of platforms profitably operating in the market—with platforms offering fewer candidates charging higher access fees.

These results bear some resemblance to the result in Damiano and Li (2007). They show how different types of agents self-select into different "meeting places," where they meet similar agents.

[20]We do not exclude existence of other equilibria.

The tool of separation between the meeting places is the price: Only some types find it worthwhile to pay higher price. In both their and our papers, meeting agents of similar type increases the efficiency of matching. The model in Damiano and Li (2007) differs in many assumptions from our model (see Section 5 for discussion). Most importantly, they do not investigate the network effects. In every meeting place every agent meets exactly one candidate. In our result there are two effects. One—the self-selection—is the same as described by Damiano and Li (2007), but the other—preferences over the number of candidates—is not captured by their model.

5 Extensions

This section focuses on extending the main findings of the model by relaxing the assumptions made and discussing the significance of these assumptions for the result.

5.1 Tentative Offers

The main model assumes that agents can make only a single offer. The goal of this assumption is to reflect the fact that people are able to pursue only limited number of possible relationships. Obviously, such an extreme assumption is not a realistic one. However, this section shows that the qualitative results of the model hold for some other, more realistic, offer-making procedures.

In this section, we analyze a two-step offer-making procedure for an environment where $G \sim U[0,1]$. After the agents meet their candidates and observe how much they like them, they proceed to making offers. In the first stage they can send a fixed number of *tentative offers*. Simultaneously, other agents send their tentative offers. Every agent observes the tentative offers he or she has received, before sending up to a one final offer in the second stage. The final offers are also sent simultaneously. As before, only if the final offer is reciprocated, the relationship is formed. Otherwise, both agents remain unmatched.

We assume that if agents are indifferent between sending an offer (tentative or final) or not, they do not send it. This eliminates a possible situation when agents send tentative offers to candidates that they like less than being alone, but are sure to be rejected by.

We show here that even with the two-step offer-making procedure there are limits to network effects. Adding tentative offer to the procedure increases the overall probability of getting matched.

However, when the number of the tentative offers allowed is constant, but the number of candidates increases, agents with lower utility of being alone prefer markets with fewer candidates, while agents with higher utility of being alone prefer markets with more candidates. A fixed number of tentative offers reflects in a more realistic way the limitations to how many potential relations people can pursue.[21] This section illustrates this point through an example of the market with two tentative offers allowed. However, the same holds for any fixed number of tentative offers.

Consider an equilibrium where every agent makes the tentative offers to his two best candidates, provided that at least two candidates are above the reservation threshold. Otherwise, the agent makes a tentative offer to the best candidate — if the best candidate is above the reservation threshold — or to no candidates, if no candidates are above the threshold. If an agent got a tentative offer from his best candidate, he makes the final offer to this candidate. If an agent did not get a tentative offer from the best candidate, but got one from the second-best candidate, then the agent makes the final offer to the second-best candidate. If the agent did not get a tentative offer either from the best or the second-best candidate, he does not make a final offer and remains unmatched.[22]

For the purpose of the comparative statics we are looking for, we need to find the expected payoff of agent $\mathfrak{a}$ when everyone meets $N \geq 2$ candidates. An agent gets a tentative offer from a particular candidate when he is either the first or the second choice of this candidate, and he is above the candidate's reservation value. An agent is the first choice of a candidate (and above the reservation value) with a probability

$$Pr(best|N) = \frac{1}{N+1}\,.$$

[21] In a labor market, it reflects the fact that an agent can go to only a limited number of interviews. In the case of an auction site, an agent can follow only a limited number of ongoing auctions.

[22] There are also other equilibria possible. All have the following structure: Let Λ^{MAX} be the Λ of the best candidate. If agent $\mathfrak{a}$ got a tentative offer from a candidate whose Λ is at least $x(\Lambda^{MAX})$, he makes the final offer to the best of such candidates, even if he did not make a tentative offer to this candidate. If the agent did not get a tentative offer from any of the candidates above $x(\Lambda^{MAX})$, he makes the final offer to his best candidate, even though he did not received a tentative offer from this candidate. The additional probability of successfully matching in such equilibrium is very small and decreasing with the number of candidates. Therefore, it does not change the qualitative results of this section.

An agent is the second choice of a candidate (and above the reservation value) with a probability

$$Pr(2nd|N) = \int_0^1 \int_{\mathfrak{a}}^1 (N-1)(1-\Lambda)\Lambda^{N-2} d\Lambda d\mathfrak{a} = \frac{N-1}{N(N+1)} \,.$$

Thus, the probability that the agent gets a tentative offer from a particular candidate is

$$Pr(tentative|N) = Pr(best|N) + Pr(2nd|N) = \frac{2N-1}{N(N+1)} \,.$$

An agent makes the final offer to the best candidate when he got a tentative offer from this candidate, and he likes the candidate more than being alone. However, it may be that the agent has not got a tentative offer from the best candidate, but he got one from the second-best candidate. If this is the case, and the second-best candidate is above the reservation threshold, the agent makes the final offer to the second-best candidate.

The agent gets the final offer when he is the most preferred candidate, or when he is the second-best candidate, but the best candidate did not make a tentative offer. Moreover, the agent gets the final offer from a candidate only if both he and the candidate made tentative offers to each other. The probability that the candidate makes tentative offer is already incorporated in the probability of getting the final offer. But we need to remember that the agent makes a tentative offer to the best or the second-best candidate only if the candidate is above the reservation value $\mathfrak{a}$. That is, the probability of getting both the tentative and the final offers is

$$\begin{aligned}
&\Big[Pr(best|N) + Pr(2nd|N)\cdot\big(1 - Pr(tentative|N)\big)\Big]\cdot Pr(\text{candidate above } \mathfrak{a}) = \\
&= \Big[\underbrace{Pr(tentative|N)\cdot\big(1 - Pr(2nd|N)\big)}_{Pr(final|N)}\Big]\cdot Pr(\text{candidate above } \mathfrak{a}) = \\
&= \frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\cdot Pr(\text{candidate above } \mathfrak{a})\,.
\end{aligned}$$

For the future reference, it is useful to define $Pr(final|N) = Pr(tentative|N)\cdot\big(1 - Pr(2nd|N)\big)$.

The agent matches with the best candidate when he received a tentative and the final offers from that candidate and the candidate was better than being alone. The probability that the best candidate out of N is above $\mathfrak{a}$ is $1-\mathfrak{a}^N$. Therefore, the agent matches with the best candidate with probability

$$Pr(match\ best|N,\mathfrak{a}) = \frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\left(1-\mathfrak{a}^N\right)\,.$$

The agent matches with the second best candidate when he received a tentative and final offer from that candidate, the second-best candidate was better than being alone, and he did not receive a tentative offer from the best candidate. The probability that the second-best candidate is above $\mathfrak{a}$ is

$$N(N-1)\int_{\mathfrak{a}}^{1}\Lambda^{N-2}(1-\Lambda)d\Lambda = 1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\,.$$

Thus, the agent matches with the second-best candidate with probability

$$\begin{aligned}Pr(match\ 2nd|N,\mathfrak{a}) &= \big(1-Pr(tentative|N)\big)\cdot Pr(final|N)\cdot\big(1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big) = \\ &= \left(1-\frac{2N-1}{N(N+1)}\right)\frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\cdot\big(1-\mathfrak{a}^N - N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\ .\end{aligned}$$

With the remaining probability of

$$\begin{aligned}&1-Pr(match\ best|N,\mathfrak{a})-Pr(match\ 2nd|N,\mathfrak{a}) = \\ &\quad = 1-Pr(final|N)\Big(1-\mathfrak{a}^N+\big(1-Pr(tentative|N)\big)\big(1-\mathfrak{a}^N-N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\Big) = \\ &\quad 1-\frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\left(1-\mathfrak{a}^N+\left(1-\frac{2N-1}{N(N+1)}\right)\big(1-\mathfrak{a}^N-N\cdot\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\right)\end{aligned}$$

the agent remains unmatched and receives the payoff of $\mathfrak{a}$.

The expected payoff from matching with the best candidate out of N is

$$EU(match\ best|\mathfrak{a},N) = N\int_{\mathfrak{a}}^{1}\Lambda^N dN = \frac{N}{N+1}\left(1-\mathfrak{a}^{N+1}\right)\ .$$

Notice that this formula already accounts for probability that the best candidate is above $\mathfrak{a}$.

The expected payoff from matching with the second-best candidate out of N is

$$EU(match\ 2nd|\mathfrak{a},N) = N(N-1)\int_{\mathfrak{a}}^{1}\Lambda^{N-1}(1-\Lambda)d\Lambda = \frac{N-1}{N+1}-(N-1)\mathfrak{a}^N\left(1-\frac{N}{N+1}\mathfrak{a}\right)\ .$$

Therefore, the expected payoff for agent $\mathfrak{a}$ in a market where two tentative offers are allowed and there are N candidates is

$$\begin{aligned}&EU(\mathfrak{a}|N) = \\ &= Pr(final|N)\cdot EU(match\ best|\mathfrak{a},N)+\big(1-Pr(tentative|N)\big)Pr(final|N)\cdot EU(match\ 2nd|\mathfrak{a},N)+ \\ &\quad + \mathfrak{a}\cdot\Big[1-Pr(final|N)\big(1-\mathfrak{a}^N+\big(1-Pr(tentative|N)\big)\big(1-\mathfrak{a}^N-N\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\Big] = \\ &\quad = \frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\frac{N}{N+1}\left(1-\mathfrak{a}^{N+1}\right)+ \\ &\quad + \left(1-\frac{2N-1}{N(N+1)}\right)\frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\cdot\left(\frac{N-1}{N+1}-(N-1)\mathfrak{a}^N\left(1-\frac{N}{N+1}\mathfrak{a}\right)\right)+ \\ &\quad + \mathfrak{a}\left[1-\frac{2N-1}{N(N+1)}\cdot\frac{N^2+1}{N(N+1)}\left(1-\mathfrak{a}^N+\left(1-\frac{2N-1}{N(N+1)}\right)\big(1-\mathfrak{a}^N-N\mathfrak{a}^{N-1}(1-\mathfrak{a})\big)\right)\right]\ .\end{aligned}$$

By graphing this formula for $N \geq 2$, we see that all agents prefer $N = 3$ to $N = 2$. But the agents are divided whether they prefer 3 or 4 candidates. Agents with $\mathfrak{a} < 0.1379$ (approximately) prefer 3 candidates and agents above that thresholds prefer 4. Similarly, agents with $\mathfrak{a} < 0.3739$ prefer 4 candidates and agents above that prefer 5. In a similar way as in the basic model, it can be shown that the optimal number of candidates is weakly increasing with the utility of being alone.

Interestingly, if there is no limit on tentative offers (i.e., one can always make tentative offers to all candidates above the reservation value, as the number of candidates increases), then the probability of matching with someone above the reservation value increases with the number of candidates. There is no trade-off, and all agents always prefer to meet more candidates.

A matching platform in the market with two tentative offers. We extend the analysis of this environment to illustrate that the results for a market with a platform also apply to the case of tentative offers: For sufficiently large number of candidates in the outside market, a platform offering fewer candidates attracts agents with $\mathfrak{a}$ lower than some threshold $\mathfrak{a}^*$. Agents above the threshold stay in the outside market.

Suppose that in the environment where agents can make up to two tentative offers, there is a matching platform. We assume that agents can make the same number of tentative offers in the platform and in the outside market, but the platform differs from the outside market in the number of candidates it offers. In the outside market the agents meet Ω candidates. The platform offers fewer candidates, $N < \Omega$, and charges a positive fee f.

Agents decide whether to participate in the platform at fee f or to stay outside by comparing their expected payoff. The derivations of the expected payoffs are similar to those above. However, the presence of the platform in the market significantly changes the probabilities of matching with the best and the second-best candidate, $\Pr(best)$ and $\Pr(2nd)$. This affects the probabilities of getting a tentative offer, $\Pr(tentative)$, as well as $\Pr(final)$. Those values are affected because they depend not only on then number of candidates, but also on the types of the candidates. As different types of agents self-select to participate in the platform or stay outside, this affects the probabilities $\Pr(best)$ and $\Pr(2nd)$. Conversely, the expected payoffs of matching with the best and second-best candidate, $EU(match\ best)$ and $EU(match\ 2nd)$, as well as the probabilities that the best and second-best candidates are above $\mathfrak{a}$, are affected only by the number of candidates. Those values do not depend on the presence of multiple platforms in the market.

Given that agents with lower $\mathfrak{a}$'s prefer fewer candidates, we expect that only agents with $\mathfrak{a}$'s below some threshold $\mathfrak{a}^*$ decide to join the platform. The probabilities of getting a tentative or final offer depend on the distribution of $\mathfrak{a}$ among the participants of the platform. The probability of being the best choice of a candidate is

$$\Pr(best|\mathrm{M_N}) = \frac{1}{N}\left(1 - \frac{1}{N+1}(\mathfrak{a}^*)^N\right).$$

The probability of being the 2nd-best choice of a candidate is

$$\Pr(2nd|\mathrm{M_N}) = \frac{\int_0^{\mathfrak{a}^*}\int_{\mathfrak{a}}^1 (N-1)(1-\Lambda)\Lambda^{N-2}d\Lambda d\mathfrak{a}}{\int_0^{\mathfrak{a}^*} d\mathfrak{a}} = \frac{1}{N}\left(1 - (\mathfrak{a}^*)^{N-1} + \frac{N-1}{N+1}(\mathfrak{a}^*)^N\right).$$

Similarly, in the outside market, the probability of being the best choice of a candidate is

$$\Pr(best|\mathrm{OUT}_\Omega) = \frac{\int_{\mathfrak{a}^*}^1\int_{\mathfrak{a}}^1 \Lambda^{\Omega-1}d\Lambda d\mathfrak{a}}{\int_{\mathfrak{a}^*}^1 d\mathfrak{a}} = \frac{1}{\Omega}\left(1 - \frac{1}{\Omega+1}\cdot\frac{1-(\mathfrak{a}^*)^{\Omega+1}}{1-\mathfrak{a}^*}\right).$$

And the probability of being the 2nd-best choice of a candidate is

$$\Pr(2nd|\mathrm{OUT}_\Omega) = \frac{\int_{\mathfrak{a}^*}^1\int_{\mathfrak{a}}^1 (\Omega-1)(1-\Lambda)\Lambda^{\Omega-2}d\Lambda d\mathfrak{a}}{\int_{\mathfrak{a}^*}^1 d\mathfrak{a}} = \frac{\frac{\Omega-1}{\Omega(\Omega+1)}\left(1-(\mathfrak{a}^*)^{\Omega+1}\right) - \frac{1}{\Omega}\mathfrak{a}^*\left(1-(\mathfrak{a}^*)^{\Omega-1}\right)}{1-\mathfrak{a}^*}.$$

The expected payoff of agent with $\mathfrak{a}$ is calculated based on the same formula:

$$\begin{aligned}EU(\mathfrak{a}|\,X) = \Pr(final)\cdot EU(match\ best|\mathfrak{a},X) + \\ + \left(1-\Pr(tentative)\right)\Pr(final)\cdot EU(match\ 2nd|\mathfrak{a},X) + \\ + \mathfrak{a}\cdot\left[1-\Pr(final)\Big(1-\mathfrak{a}^X + \left(1-\Pr(tentative)\right)\left(1-\mathfrak{a}^X - X\mathfrak{a}^{X-1}(1-\mathfrak{a})\right)\Big)\right].\end{aligned}$$

The expected payoff for $\mathfrak{a}$ in the platform is obtained by substituting $N = X$ and the appropriate probabilities. Similarly, the expected payoff for $\mathfrak{a}$ in the outside market is obtained by substituting $\Omega = X$ and the appropriate probabilities. In this way, we obtain $EU(\mathfrak{a}|\mathrm{M_N})$ and $EU(\mathfrak{a}|\mathrm{OUT}_\Omega)$ as functions of $\mathfrak{a}$, N and Ω. It is worth noting that some of the formulas are obtained assuming $N \geq 2$, and should not be applied to $N = 1$.

Given the threshold, every agent compares these two expected payoffs, and decides whether to join the platform at fee f or not. In the equilibrium, the agent with $\mathfrak{a} = \mathfrak{a}^*$ is indifferent between joining at fee f or staying outside:

$$EU(\mathfrak{a}^*|\mathrm{M_N}) - f = EU(\mathfrak{a}^*|\mathrm{OUT}_\Omega).$$

For any values of N and Ω, this equation provides a relation between the fee f and the threshold $\mathfrak{a}^*$. In the case when f as a function of $\mathfrak{a}^*$ is strictly monotonic for $\mathfrak{a}^* \in [0, 1]$, there is exactly one possible threshold $\mathfrak{a}^*$ for each level of f. Examining the examples, we observe that for $N = 2$ or when $N > \Omega$, the monotonicity sometimes does not hold. However, if $N > 2$ and $\Omega > N$, f is strictly decreasing in $\mathfrak{a}^*$ on the interval $[0, 1]$.

When $f(\mathfrak{a}^*)$ is strictly decreasing, for every fee within a range, all agents with $\mathfrak{a} < \mathfrak{a}^*$ prefer to join the platform and pay the fee. And all agents with $\mathfrak{a} > \mathfrak{a}^*$ prefer to stay outside. Moreover, given the unique response of the market to each fee, the platform chooses a fee (indirectly choosing the threshold $\mathfrak{a}^*$) to maximize the profit.

Therefore, for markets with two tentative offers, it is also true that a platform offering fewer candidates than the outside market is attractive for some parts of the market, and achieves positive profits. And the result holds for any number of tentative offers, as long as the number of tentative offers is fixed and lower than the number of candidates available.

5.2 Heterogeneous value of being alone, $\mathfrak{a}$

Many papers in matching literature (e.g. Damiano and Li, 2007, 2008) assume that agents receive 0 if they remain unmatched. Sometimes this assumption is relaxed by allowing agents to receive some other value when unmatched, but this value is usually assumed the same for all agents. However, in many markets (including dating or labor markets) agents differ by the payoff they obtain when unmatched. It is not a trivial assumption, since equilibria in the market change when we allow agents to differ in their utility of being alone.

Suppose that in our model the value of being alone is 0 for all agents. Then every agent prefers the market with as little competition as possible, and therefore as few candidates as possible. A market with more candidates and more competition increases the probability of being rejected and staying alone. With the payoff of being alone 0, the increase in the expected value of the best candidate does not offset the increased probability of being rejected.

The assumption of the utility of being alone equal to 0 is an extreme assumption. Suppose that the value of being alone is some $\tilde{\mathfrak{a}}$ from the interval $(0, 1)$, but that it is the same for all agents. Since agents are all the same when they make a decision whether to join the platform, they all make the same decision. For some values of parameters Ω, N and $\tilde{\mathfrak{a}}$ there exists an equilibrium with active matching platform. In this equilibrium all agents join the matching platform. There always exists an equilibrium where no agent joins the platform. There are no other equilibria. Specifically, there does not exists an equilibrium in which part of agents strictly prefers to participate in the platform and other agents prefer to stay in the outside market.

5.3 Subjective value of a candidate, Λ

In most of the papers on matching (e.g., Damiano and Li, 2007, 2008; McAfee, 2002) agents are endowed with an attribute or attributes that are similarly desired by all potential partners. Usually such an attribute is objective "quality." However, in the labor market employers may value differently different characteristics of employees. Even more so in the dating market, where the taste for the partners is idiosyncratic.

In our model we adopt a completely opposite assumption: the values of Λ are independent. That means that when two men meet a woman, the extent to which one man likes the woman is independent of how much the other man likes her. Such an assumption, applied to the whole market in its pure form, is also not a realistic assumption. However, it gives us opportunity to study the other extreme of the market.

Moreover, such an assumption seems more realistic if applied to a selection from a market. Suppose that agents in the market have objective and subjective characteristics. Once they are separated according to their objective characteristics (e.g., education), the agents are different only according to subjective characteristics. Then the assumption of our model applies.

This interpretation illustrates that most markets involve a mixture of the two extremes. While there is a lot of research investigating one extreme, our paper investigates the other extreme. A natural extension of this research would be investigation of intermediate case, where agents' valuations are correlated, but not identical.

The assumption of subjective valuations is important in models that want to relate to reality, because whether we assume independent or perfectly correlated Λ's changes prediction of the model: For comparison, assume in our model that Λ is an objective quality of an agent, and it is how much utility his partner gets if matched — no matter who the partner is. It turns out that in such a case, all agents prefer to meet more candidates than fewer: The impact of competition is much smaller than the benefit of having more choice. This is because when meeting candidates, an agent knows not only how much he values them, but also how other agents value the same candidates, and how the candidates value his own quality. In such a case, every agent makes an offer to the candidate as close to his own quality as possible. If he makes an offer to a candidate of a higher quality, he significantly decreases the probability of having the offer reciprocated. If he makes an offer to a candidate of a lower quality, he decreases the payoff from matching. The expected payoff from making an offer is larger, the closer the candidate is to agent's own quality. Meeting larger number of candidates increases chances of meeting a candidate closer to the agent's own quality. Thus, all agents prefer a market with more candidates to a market with fewer candidates.

5.4 Correlation

6 Discussion and Conclusions

Theoretical literature on network effects suggests that offering participants access to a broader set of agents allows them to find better matches. However, in practice, we observe that platforms that restrict choice exist and prosper alongside platforms that offer more choice. Furthermore, platforms restricting choice are often able to charge higher prices. We propose a model that explains these empirical regularities. Platforms that offer fewer choices than their competitors exist because they in markets with heterogeneous agents, they can attract different types of agents. Accounting for such heterogeneity allows us to explain not only why two platforms can coexist without the market tipping, but also why an industry may experience entry of a firm with a seemingly inferior product. In fact, we show that a firm that offers such a product (restricted choice) may charge higher prices and be more profitable than its competitors who offer unrestricted choice.

While our paper captures the stylized facts of the online dating industry, it also delivers a series of additional empirical predictions. Our model predicts that the demand for platform services is non-monotonic in the number of candidates that the platform offers. Moreover, restricted-choice platforms should have a higher probability of transaction occurring (or lower expected time to transaction occurring). An important feature of our model is that when agents choose a platform, they self-select based on their characteristics. Platforms that restrict choice will appeal primarily to agents who are impatient or who have lower outside options. Agents with more patience or better outside option will use platforms that maximize choice. The higher the difference in fees charged, the greater will be the differences between participants of the different platforms. This suggests that empirical methodology used in settings with competing platforms should be robust to such realized heterogeneity.

Finally, our analysis has implications for managers seeking to enter into or compete in industries with strong network effects. While prevailing wisdom suggests that offering a large choice set to consumers on their platforms should benefit all consumers, our model shows that this intuition may not always hold. We discuss two important dimensions to consider before deciding how to compete. Specifically, the received wisdom holds in markets where people do not differ much in their utility of being unmatched, or where preferences are fairly homogeneous across agents. However, when people differ dramatically in their outside options, and when preferences are highly subjective, managers have more flexibility in how to compete and may want to enter the market as a restricted-choice platform.

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Appendix

Proof of Lemma 1:

(i) With N candidates, a woman that the man meets has $N+1$ possible actions: to make an offer to one of the N candidates and to make no offer (when $\mathfrak{a}^w$ is larger than any of the relevant Λ's). All Λ's and $\mathfrak{a}^w$ are drawn independently from the same distribution. Therefore, without knowing $\mathfrak{a}^w$, each of the actions is equally likely.

(ii) The agent makes an offer to the best Λ, if the highest Λ is above $\mathfrak{a}$. The highest Λ is above $\mathfrak{a}$ with probability $1 - G^N(\mathfrak{a})$. Independently, the best Λ makes an offer to agent $\mathfrak{a}$ with probability $\frac{1}{N+1}$ (from point (1) of this Lemma).

(iii) Unconditional expected value of a match is $pr(accepted) \cdot E(\max\Lambda | \max\Lambda > \mathfrak{a})$. Thus, the value of matching, conditional on being accepted is $E(\max\Lambda | \max\Lambda > \mathfrak{a})$.

To find the conditional expected value of $E(\max\Lambda | \max\Lambda > \mathfrak{a})$, we first characterize the distribution function of $\max\Lambda$ under N candidates. Notice that the cdf of $\max\Lambda$ is $Pr(\max\Lambda < x) = G^N(x)$. Thus, the pdf is $\frac{\partial G^N(x)}{\partial x} = NG^{N-1}(x)g(x)$. Using the probability density, we calculate the expected value of $\max\Lambda$, given that $\max\Lambda > \mathfrak{a}$:

$$\int_{\mathfrak{a}}^{1} NG^{N-1}(x)g(x) \cdot xdx = N\int_{\mathfrak{a}}^{1} G^{N-1}(x)g(x)xdx\,.$$

(iv) Follows directly from parts (i), (ii) and (iii) of the Lemma.

This completes the proof of Lemma 1. $\square$

Proof of Lemma 2: Using integration by parts,

$$N\int_{\mathfrak{a}}^{1} G^{N-1}(x)g(x)xdx = G^N(x)x\big|_{\mathfrak{a}}^{1} - \int_{\mathfrak{a}}^{1} G^N(x)dx =$$
$$= G^N(1)\cdot 1 - G^N(\mathfrak{a})\mathfrak{a} - \int_{\mathfrak{a}}^{1} G^N(x)dx = 1 - G^N(\mathfrak{a})\mathfrak{a} - \int_{\mathfrak{a}}^{1} G^N(x)dx$$

Since $G(x) \le 1$ for $0 \le x \le 1$, $G^N(x)$ is nonincreasing with N for $0 \le x \le 1$ and $\int_{\mathfrak{a}}^{1} G^N(x)dx$ is nonincreasing with N, $1 - G^N(\mathfrak{a})\mathfrak{a} - \int_{\mathfrak{a}}^{1} G^N(x)dx$ is nondecreasing with N. $\square$

Lemma A.1 *Consider an arbitrary* $\hat{\mathfrak{a}} \in [0, 1)$.

(i) When $EU(\hat{\mathfrak{a}}|N+1) - EU(\hat{\mathfrak{a}}|N) \geq 0$, *then for all* $\mathfrak{a} \in (\hat{\mathfrak{a}}, 1)$, $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) > 0$.

(ii) When $EU(\hat{\mathfrak{a}}|N+1) - EU(\hat{\mathfrak{a}}|N) \leq 0$, *then for all* $\mathfrak{a} \in [0, \hat{\mathfrak{a}})$, $EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) < 0$.

Proof of Lemma A.1: Notice that

$$EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N) = \frac{1}{(N+1)(N+2)} \int_a^1 \left[G^N(x) - 1 + (N+1)G^N(x)(1 - G(x))\right] dx\,.$$

Let's identify the sign of $\int_a^1 F(x)dx$, where

$$F(x) = G^N(x) - 1 + (N+1)G^N(x)(1 - G(x))\,.$$

It is useful to learn the shape of $F(x)$ to determine the sign of $\int_a^1 F(x)dx$. For $x = 0$, $F(x) = -1$, and for $x = 1$, $F(x) = 0$. Moreover, it is single peaked: increasing for $x < \hat{x}$ and decreasing for $x > \hat{x}$, with maximum at $\hat{x}$ s.t. $G(\hat{x}) = \frac{(N+1)^2-1}{(N+1)^2}$.

Since for $x \in (\hat{x}, 1]$, $F(x)$ decreases and $F(1) = 0$, then $F(\hat{x}) > 0$. Moreover, $F(0) = -1$ and for $x \in [0, \hat{x})$, $F(x)$ increases. Therefore, $\exists \hat{\hat{x}} \in (0, \hat{x})$ s.t. $F(\hat{\hat{x}}) = 0$.

Now, suppose $\int_{\hat{a}}^1 F(x)dx \geq 0$. Take $a > \hat{a}$. Then

$$\int_a^1 F(x)dx = \int_{\hat{a}}^1 F(x)dx - \int_a^{\hat{a}} F(x)dx\,.$$

If $a > \hat{\hat{x}}$, then $F(x) > 0$ for all $x > a$, so $\int_a^1 F(x)dx > 0$. If $a \leq \hat{\hat{x}}$, then $F(x) < 0$ for all $x \in [\hat{a}, a)$, so $\int_{\hat{a}}^a F(x)dx < 0$ and

$$\int_a^1 F(x)dx = \int_{\hat{a}}^1 F(x)dx - \int_{\hat{a}}^a F(x)dx > \int_{\hat{a}}^1 F(x)dx\,.$$

For the second part of the lemma, suppose that $\int_{\hat{a}}^1 F(x)dx \leq 0$, and take $a < \hat{a}$. For $\int_{\hat{a}}^1 F(x)dx \leq 0$ it must be that $\hat{a} < \hat{\hat{x}}$. This is because for all $y > \hat{\hat{x}}$, $\int_y^1 F(x)dx > 0$. Then $\int_a^{\hat{a}} F(x)dx < 0$, and so $\int_a^1 F(x)dx < 0$. This completes the proof of the lemma. $\square$

Proof of Proposition 4: Let $\Delta(\mathfrak{a}|N) = EU(\mathfrak{a}|N+1) - EU(\mathfrak{a}|N)$

Step 1. $F(x)$ is decreasing in N, for any x. We show that $F(x,N) - F(x,N+1) > 0$.

$$G^N(x) - 1 + (N+1)G^N(x)(1-G(x)) - G^{N+1}(x) + 1 - (N+2)G^{N+1}(x)(1-G(x)) =$$
$$= G^N(x)(1-G(x)) + (N+1)G^N(x)(1-G(x)) - (N+2)G^{N+1}(x)(1-G(x)) =$$
$$= (N+2)G^N(x)(1-G(x)) - (N+2)G^{N+1}(x)(1-G(x)) =$$
$$= (N+2)G^N(x)(1-G(x))(1-G(x)) > 0$$

Step 2. $\Delta(\mathfrak{a}|N)$ is decreasing in N, for any $\mathfrak{a}$. The fact that $F(x,N+1) < F(x,N)$ may not be enough to prove

$$\underbrace{\Delta(\mathfrak{a}|N+1)}_{=\int_a^1 \frac{F(x,N+1)}{(N+2)(N+3)}dx} < \underbrace{\Delta(\mathfrak{a}|N)}_{=\int_a^1 \frac{F(x,N)}{(N+1)(N+2)}dx}$$

However, we can show that for any x

$$\frac{F(x,N+1)}{(N+2)(N+3)} < \frac{F(x,N)}{(N+1)(N+2)}\,,$$

because $F(x,N+1) < F(x,N)$ and $\frac{1}{N+3} < \frac{1}{N+1}$. Since at every point the integrated function is smaller, the integral also needs to be smaller. Alternatively:

$$\Delta(a|N+1) - \Delta(a|N) = \int_a^1 \frac{F(x,N+1)}{(N+2)(N+3)}dx - \int_a^1 \frac{F(x,N)}{(N+1)(N+2)}dx =$$
$$= \int_a^1 \left(\frac{F(x,N+1)}{(N+2)(N+3)} - \frac{F(x,N)}{(N+1)(N+2)}\right) dx < 0\,,$$

because at any point x the integrated function is negative.

Step 3. For any $\mathfrak{a}$, there exists finite N such that $\Delta(\mathfrak{a}|N) < 0$. Suppose that $\Delta(\mathfrak{a}|1) > 0$ (otherwise $\bar{N}(\mathfrak{a}) = 1$ and the lemma is satisfied). For every $x \in (0,1)$, $F(x) \to_{N\to\infty} -1$. Hence, as N goes to infinity, $\int_a^1 F(x)dx \to -(1-a) < 0$. Then, there must be an N such that $\Delta(\mathfrak{a}|N) < 0$. Let $\bar{N}(\mathfrak{a})$ be the smallest N such that $\Delta(\mathfrak{a}|N) < 0$. Therefore, for every $\mathfrak{a}$ there exists such $\bar{N}(\mathfrak{a})$. From *Step 2.*, we know that $\Delta(\mathfrak{a}|N)$ is (strictly) decreasing in N for any $\mathfrak{a}$. Therefore, for any $N < \bar{N}(\mathfrak{a})$, $\Delta(\mathfrak{a}|N)$ is positive, and for any $N \geq \bar{N}(\mathfrak{a})$ it is negative.

Step 4. $\bar{N}(\mathfrak{a})$ is non-decreasing in $\mathfrak{a}$. For any $\mathfrak{a}'$ and $\mathfrak{a}''$ such that $\mathfrak{a}'' > \mathfrak{a}'$, we show that $\bar{N}(\mathfrak{a}'') \geq \bar{N}(\mathfrak{a}')$. Let $N' \equiv \bar{N}(\mathfrak{a}')$. That is $\Delta(\mathfrak{a}'|N) > 0$ for $N < N'$ and $\Delta(\mathfrak{a}'|N) < 0$ for $N \geq N'$. Now consider $\mathfrak{a}'' > \mathfrak{a}'$. According to previous lemma, when $\Delta(\mathfrak{a}'|N) > 0$, then $\Delta(\mathfrak{a}''|N) > 0$. Therefore, for $N < N'$, $\Delta(\mathfrak{a}''|N) > 0$. Since for every $\mathfrak{a}$ there exists $\bar{N}(\mathfrak{a})$ (*Step 3*), it must be that for $\mathfrak{a}''$, $\bar{N}(\mathfrak{a}'') \geq N'$. $\square$

Lemma A.2 *For $\Omega > N$, $\frac{G^N(a)-G^\Omega(a)}{1-G^N(a)}$ is strictly increasing on $\mathfrak{a} \in (0,1)$.*

Proof of Lemma A.2: Consider only $\mathfrak{a} \in [0,1]$. Let $x = \Omega - N > 0$. The derivative of $\frac{G^N(a)-G^\Omega(a)}{1-G^N(a)} = \frac{G^N(a)-G^{N+x}(a)}{1-G^N(a)}$ with respect to $\mathfrak{a}$ is then

$$\frac{[NG^{N-1}(a)g(a) - (N+x)G^{N+x-1}(a)g(a)](1-G^N(a)) + NG^{N-1}(\mathfrak{a})g(\mathfrak{a})G^N(\mathfrak{a})(1-G^x(\mathfrak{a}))}{(1-G^N(a))^2} =$$

$$= \frac{G^{N-1}(a)g(a)}{(1-G^N(a))^2}\left[[N-(N+x)G^x(a)](1-G^N(a)) + NG^N(a)(1-G^x(a))\right] =$$

$$= \underbrace{\frac{G^{N-1}(a)g(a)}{(1-G^N(a))^2}}_{+}\underbrace{[N-(N+x)G^x(a)+xG^{N+x}(a)]}_{X(N,x,\mathfrak{a})}$$

The sign of the derivative is the same as the sign of $X(N,x,\mathfrak{a})$. We claim that $X(N,x,\mathfrak{a})$ is positive. First, notice that for $a=1$, $X(N,x,1)=0$. Moreover, the derivative of $X(N,x,\mathfrak{a})$ with respect to $\mathfrak{a}$ is negative:

$$\underbrace{x(N+x)}_{>0}\underbrace{G^{x-1}(a)g(a)}_{+}\underbrace{(G^N(a)-1)}_{-} < 0\,.$$

This is enough to establish that $X(N,x,\mathfrak{a})$ is positive. In addition notice that $X(N,x,\mathfrak{a}=0) = N > 0$.

Therefore $\frac{G^N(a)-G^\Omega(a)}{1-G^N(a)}$ is strictly increasing for $\Omega > N$. $\square$

Proof of Lemma 5: Agent $\mathfrak{a}$'s willingness to pay to join platform N is equal to the additional expected payoff that the agent can get by joining the platform, i.e., $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N) - EU(\mathfrak{a}|\Omega)$, where

$$EU(\mathfrak{a}|N) = [G^N(\mathfrak{a}) + (1-G^N(\mathfrak{a}))Pr(rej|N)]\mathfrak{a} + (1-Pr(rej|N)) \cdot N\int_a^1 G^{N+1}(x)g(x)xdx$$

$$= 1 + Pr(rej|N)(\mathfrak{a}-1) - (1-Pr(rej|N))\int_a^1 G^N(x)dx$$

$$EU(\mathfrak{a}|\Omega) = [G^\Omega(\mathfrak{a}) + (1-G^\Omega(\mathfrak{a}))Pr(rej|\Omega)]\mathfrak{a} + (1-Pr(rej|\Omega)) \cdot N\int_a^1 G^{\Omega+1}(x)g(x)xdx$$

$$= 1 + Pr(rej|\Omega)(\mathfrak{a}-1) - (1-Pr(rej|\Omega))\int_a^1 G^\Omega(x)dx$$

Then,

$$\begin{aligned}
&WTP(\mathfrak{a}) = EU(\mathfrak{a}|N) - EU(\mathfrak{a}|\Omega) = \\
&= (1-\mathfrak{a})[Pr(rej|\Omega) - Pr(rej|N)] - [1 - Pr(rej|N)] \int_a^1 G^N(x)dx + [1 - Pr(rej|\Omega)] \int_a^1 G^\Omega(x)dx = \\
&= [1 - Pr(rej|\Omega)] \int_a^1 [G^\Omega(x) - G^N(x)]dx + [Pr(rej|\Omega) - Pr(rej|N)] \cdot \left(1 - a - \int_a^1 G^N(x)dx\right)
\end{aligned}$$

Notice that for $\mathfrak{a} = 1$, $WTP(\mathfrak{a} = 1) = 0$; for $\mathfrak{a} = 0$, WTP may be positive or negative.

$$\frac{\partial WTP(\mathfrak{a})}{\partial \mathfrak{a}} = (1 - Pr(rej|\Omega)) \cdot [G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a})] - (Pr(rej|\Omega) - Pr(rej|N)) \cdot (1 - G^N(\mathfrak{a}))$$

$$\frac{\partial WTP(\mathfrak{a})}{\partial \mathfrak{a}} < 0 \iff \frac{G^N(a) - G^\Omega(a)}{1 - G^N(a)} < \frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)}$$

We consider $N < \Omega$. Then $G^N(\mathfrak{a}) - G^\Omega(\mathfrak{a}) > 0$. From Lemma A.2 we know that $\frac{G^N(a) - G^\Omega(a)}{1 - G^N(a)}$ is strictly increasing. Moreover, it takes value 0 for $\mathfrak{a} = 0$, and $\frac{\Omega - N}{N} > 0$ as $\mathfrak{a} \to 1$.

If $Pr(rej|N) < Pr(rej|\Omega)$, then $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)}$ is a positive constant. When $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)} > \frac{\Omega - N}{N}$, the $WTP(\mathfrak{a})$ is decreasing on the whole interval $\mathfrak{a} \in [0, 1)$, and hence everywhere positive.

When $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)} < \frac{\Omega - N}{N}$, the $WTP(\mathfrak{a})$ is decreasing for small $\mathfrak{a}$'s, and increasing for large $\mathfrak{a}$'s. But since for $\mathfrak{a} = 1$, $WTP = 0$, WTP must increase to 0 from negative values. Therefore, for $\mathfrak{a}$'s where $WTP(\mathfrak{a}) > 0$, WTP is strictly decreasing. □

Corollary A.3 *For any given $Pr(rej|N)$ and $Pr(rej|\Omega) < Pr(rej|N)$ and for any Ω and $N < \Omega$, the willingness to pay $WTP(\mathfrak{a}) = EU(\mathfrak{a}|N, Pr(rej|N)) - EU(\mathfrak{a}|\Omega, Pr(rej|\Omega))$ is non-positive for $\mathfrak{a} \in [0, 1]$.*

Proof of Corollary A.3: Consider WTP from the proof of Lemma 5, and suppose $N < \Omega$. If $Pr(rej|N) > Pr(rej|\Omega)$, then $\frac{Pr(rej|\Omega) - Pr(rej|N)}{1 - Pr(rej|\Omega)} < 0$ while $\frac{G^N(a) - G^\Omega(a)}{1 - G^N(a)} > 0$. Therefore $\frac{\partial WTP(\mathfrak{a})}{\partial \mathfrak{a}} > 0$, i.e., WTP is strictly increasing everywhere. And since $WTP(\mathfrak{a} = 1) = 0$, then $WTP(\mathfrak{a})$ is negative for $\mathfrak{a} \in [0, 1)$. Thus, no agent has a positive willingness to pay to join a platform offering fewer candidates and higher probability of rejection. □

Corollary A.4 *When fee $f = 0$, then $\mathfrak{a}^*(f{=}0) = 1$. That is, all agents prefer to join the platform if the fee is the same as for participating on the outside market.*

Proof of Corollary A.4: It follows from the fact that $EU(\mathfrak{a}^*|N) - EU(\mathfrak{a}^*|\Omega)$ is positive on the interval $[0,1)$. Agents with $\mathfrak{a} \in [0,1)$ prefer to join when $f = 0$, and agents with $\mathfrak{a} = 1$ are indifferent. $\square$

Proof of Proposition 6: Let $N < \Omega$. Suppose that agents then expect $Pr(rej|N) < Pr(rej|\Omega)$. From Lemma 5, we know that in such a case, on the interval $\mathfrak{a} \in [0, \bar{\mathfrak{a}})$ for $\bar{\mathfrak{a}} < 1$ willingness to pay is positive and decreasing (and continuous). Therefore, the WTP is highest for $\mathfrak{a} = 0$, $WTP(\mathfrak{a} = 0) > 0$. Thus, for any fee $f < WTP(\mathfrak{a} = 0)$ there exists $\mathfrak{a}'$ such that $f = WTP(a')$. All agents with $a < a'$ have higher willingness to pay than f. Thus, they will pay the fee and join the platform, and the platform collects profits $a' \cdot WTP(\mathfrak{a}')$. In an equilibrium, the expectations need to be fulfilled. Since for given f agents with $a < a'$ join the platform, and those with $a > a'$ stay outside:

$$
\begin{aligned}
Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}')) &= 1 - \frac{1}{N} + \frac{G^N(a')}{N(N+1)} \\
Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}', 1]) &= 1 - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')}
\end{aligned}
$$

Notice that $Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}', 1]) > Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}'))$

$$
\begin{aligned}
Pr(rej|\Omega, \mathfrak{a} \in (\mathfrak{a}', 1]) - Pr(rej|N, \mathfrak{a} \in [0, \mathfrak{a}')) &> 0 \\
1 - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} - \left(1 - \frac{1}{N} + \frac{G^N(a')}{N(N+1)}\right) &> 0 \\
\frac{1}{N} - \frac{1}{\Omega} + \frac{1}{\Omega(\Omega+1)} \frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} - \frac{G^N(a')}{N(N+1)}) &> 0 \\
\Omega(\Omega+1)(N+1) - N(N+1)(\Omega+1) + N(N+1)\frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} - \Omega(\Omega+1)G^N(a') &> 0 \\
\Omega(\Omega+1)N - N(N+1)(\Omega+1) + N(N+1)\frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} + \Omega(\Omega+1)\left(1 - G^N(a')\right) &> 0 \\
N(\Omega+1)\underbrace{(\Omega - N - 1)}_{\geq 0} + N(N+1)\frac{1 - G^{\Omega+1}(\mathfrak{a}')}{1 - G(\mathfrak{a}')} + \Omega(\Omega+1)\left(1 - G^N(a')\right) &> 0
\end{aligned}
$$

All other terms are strictly positive. Moreover, notice that for $a = 0$ and for $a = \bar{a}$, $a \cdot WTP(a) =$. But for $\mathfrak{a} \in (0, \bar{a})$ both a and $WTP(a)$ are positive, so $a \cdot WTP(a) > 0$. Let $\mathfrak{a}^*$ be the value that maximizes platform's profits. Then it must be that $\mathfrak{a}^* \in (0, \bar{a})$ and $\mathfrak{a}^* \cdot WTP(\mathfrak{a}^*) > 0$. $\square$

Proof of Lemma 7: We obtain the result by differentiating

$$EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a} = 1 - Pr(rej|M_2)(\mathfrak{a} - 1) - (1 - Pr(rej|M_2)) \int_{\mathfrak{a}}^{1} G^{M_2}(x)dx - \mathfrak{a}$$

with respect to $\mathfrak{a}$:

$$\frac{\partial(EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a})}{\partial\mathfrak{a}} = Pr(rej|M_2) + [1 - Pr(rej|M_2)]\, G^{M_2}(\mathfrak{a}) - 1 = [1 - Pr(rej|M_2)] \underbrace{\left[G^{M_2}(\mathfrak{a}) - 1\right]}_{-} < 0\,.$$

Thus, $EU(\mathfrak{a}|M_2, Pr(rej|M_2))$ is decreasing on the whole range of $\mathfrak{a}$.

Moreover, $EU(\mathfrak{a}|M_2, Pr(rej|M_2)) - \mathfrak{a}$ evaluated at $\mathfrak{a} = 1$ is

$1 + Pr(rej|M_2) \cdot 0 - (1 - Pr(rej|M_2)) \cdot 0 - 1 = 0.$ $\square$

Proof of Proposition 9: Suppose $M_1 < M_2$. Lemma 7 and Corollary 8 help characterize the agents' decisions about which platform to join, if any, given f_1, $f_2 < f_1$, $Pr(rej|M_1)$, $Pr(rej|M_2) > Pr(rej|M_2)$.

Agents with $\mathfrak{a} < \mathfrak{a}_2^*$ prefer M_2 to being unmatched, and those with $\mathfrak{a} > \mathfrak{a}_2^*$ prefer being unmatched to M_2. Agents with $\mathfrak{a} < \mathfrak{a}_1^*$ prefer M_1 to M_2, and those with $\mathfrak{a} > \mathfrak{a}_1^*$ prefer M_2 to M_1.

If $\mathfrak{a}_1^* > \mathfrak{a}_2^*$, no agent chooses to join platform M_2.

When $\mathfrak{a}_1^* < \mathfrak{a}_2^*$, agents with $\mathfrak{a} < \mathfrak{a}_1^*$ choose M_1, agents with $\mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*)$ choose M_2, and agents with $\mathfrak{a} > \mathfrak{a}_2^*$ stay unmatched. When tis is the case, then the resulting rejection probabilities are indeed $\Pr(rej|M_1, \mathfrak{a} \in [0, \mathfrak{a}_1^*)) < Pr(rej|M_2, \mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*))$.

Thresholds $\mathfrak{a}_1^*$ and $\mathfrak{a}_2^*$ depend on f_1 and f_2, which are set by the platforms. Platforms take into account the resulting decisions of agents when setting their fees. Notice that f_1 and f_2 uniquely characterize $\mathfrak{a}_1^*(f_1, f_2)$ and $\mathfrak{a}_2^*(f_1, f_2)$. Therefore, we can think of the platforms as effectively choosing a_i^* given a_j^*.

Platforms' profits are a product of their fees and the measure of agents who join them. First, notice that platform M_1 would never set $\mathfrak{a}_1^* = 1$, as it would require $f_1 = 0$ (to attract $\mathfrak{a} = 1$), and would result in 0 profits, while positive profits for other $\mathfrak{a}_1^*$ are available. Similarly, platform M_1 never sets f_1 so high that $\mathfrak{a}_1^* = 0$, as it also results in 0 profits.

Next, notice that platform M_2 would never set $\mathfrak{a}_2^* \leq \mathfrak{a}_1^*$, as it would bring it 0 profit. Also, setting $\mathfrak{a}_2^* = 1$ would require $f_2 = 0$, and would result in 0 profits, therefore, is suboptimal for M_2.

Thus, in an equilibrium $0 < \mathfrak{a}_1^* < \mathfrak{a}_2^* < 1$. To show that such an equilibrium exists, we turn to analyzing platforms' best response curves. The profits are

$$\begin{aligned}\pi_1(\mathfrak{a}_1^*|\mathfrak{a}_2^*) =& \mathfrak{a}_1^* \cdot f_1(\mathfrak{a}_1^*, \mathfrak{a}_2^*) \\ \pi_2(\mathfrak{a}_2^*|\mathfrak{a}_1^*) =& (\mathfrak{a}_2^* - \mathfrak{a}_1^*)\, f_2(\mathfrak{a}_1^*, \mathfrak{a}_2^*)\,,\end{aligned}$$

where f_1 and f_2 are characterized by the indifference conditions

$$\begin{aligned}f_2(\mathfrak{a}_1^*, \mathfrak{a}_2^*) &= EU(\mathfrak{a}_2^*|M_2) - \mathfrak{a}_2^* \\ f_1(\mathfrak{a}_1^*, \mathfrak{a}_2^*) &= EU(\mathfrak{a}_1^*|M_1) - EU(\mathfrak{a}_1^*|M_2) + f_2 = \\ &= EU(\mathfrak{a}_1^*|M_1) - EU(\mathfrak{a}_1^*|M_2) + EU(\mathfrak{a}_2^*|M_2) - \mathfrak{a}_2^*\end{aligned}$$

The best responses $\mathfrak{a}_1^*(\mathfrak{a}_2^*)$ and $\mathfrak{a}_2^*(\mathfrak{a}_1^*)$ satisfy first order conditions[23]

$$\begin{aligned}\frac{\partial \pi_1}{\partial \mathfrak{a}_1^*} =& EU(\mathfrak{a}_1^*|M_1) - EU(\mathfrak{a}_1^*|M_2) + EU(\mathfrak{a}_2^*|M_2) - \mathfrak{a}_2^* + \mathfrak{a}_1^* \left(\frac{\partial\, [EU(\mathfrak{a}_1^*|M_1) - EU(\mathfrak{a}_1^*|M_2)]}{\partial\, \mathfrak{a}_1^*} + \frac{\partial\, EU(\mathfrak{a}_2^*|M_2)}{\partial\, \mathfrak{a}_1^*} \right) = 0 \\ \frac{\partial \pi_2}{\partial \mathfrak{a}_2^*} =& EU(\mathfrak{a}_2^*|M_2) - \mathfrak{a}_2^* + (\mathfrak{a}_2^* - \mathfrak{a}_1^*) \left(\frac{\partial\, EU(\mathfrak{a}_2^*|M_2)}{\partial\, \mathfrak{a}_2^*} - 1 \right) = 0\,.\end{aligned}$$

We don't know the exact shape of the best response curves. But we still can characterize certain aspects of them. First, consider the best response of platform M_1 to $\mathfrak{a}_2^*$ set by M_2. When $M_2 = 0$, M_1 is de facto a monopolist, where the outside option for the agents is to stay unmatched. The optimal $\mathfrak{a}_1^*(\mathfrak{a}_2^* = 0) \in (0, 1)$. When $\mathfrak{a}_2^* = 1$ (i.e., $f_2 = 0$), then M_1's situation is as in Section 4.1, with the outside market offering M_2 candidates. The optimal $\mathfrak{a}_1^*(\mathfrak{a}_2^* = 1) \in (0, 1)$ as well. Moreover, for all other values of $\mathfrak{a}_2^*$, $\mathfrak{a}_1^*(\mathfrak{a}_2^*)$ is continuous.

Next, consider the best response of M_2 to $\mathfrak{a}_1^*$. When $\mathfrak{a}_1^* = 0$, M_2 is de facto a monopolist, and the optimal $\mathfrak{a}_2^*(\mathfrak{a}_1^* = 0) \in (0, 1)$. And for $\mathfrak{a}_1^* \to 1$, $\mathfrak{a}_2^*(\mathfrak{a}_1^* \to 1) \to 1$ (because $\mathfrak{a}_1^* < \mathfrak{a}_2^*(\mathfrak{a}_1^*) < 1$). And since $\mathfrak{a}_1^*(\mathfrak{a}_2^* \to 1) < 1$, and both curves are continuous, they must intersect at least once for interior values of a_i^* (see the Figure 2). Hence an equilibrium exists. And since $\mathfrak{a}_1^* < \mathfrak{a}_2^*(\mathfrak{a}_1^*) < 1$, the inequality $\mathfrak{a}_1^* < \mathfrak{a}_2^*$ holds in this equilibrium. □

[23]Note that $\frac{\partial\, EU(\mathfrak{a}_2^*|M_2)}{\partial\, \mathfrak{a}_1^*} \neq 0$, because $\mathfrak{a}_1^*$ affects $Pr(rej|M_2, \mathfrak{a} \in (\mathfrak{a}_1^*, \mathfrak{a}_2^*))$, and the platform is aware of it when calculating its best response.

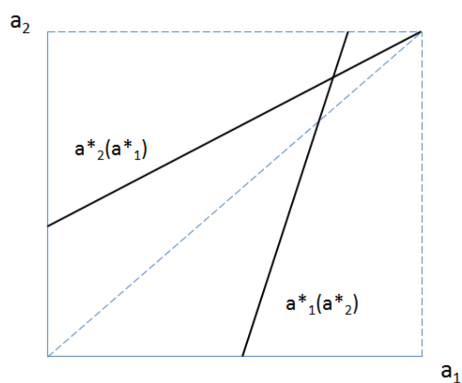

Figure 2: We don't know the exact shape of the best response curves. But by the characteristics of the "endpoints", and the fact that both best response curves are continuous (from the first order conditions of platforms' profit maximization problems), they must intersect. And since $\mathfrak{a}_2^*(\mathfrak{a}_1^*) > \mathfrak{a}_1^*$, it assures the properties of the equilibrium.