Optimal Disclosure When Some Information Is Soft*

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Abstract

This paper studies a firm's optimal disclosure policy when some information ("soft") cannot be disclosed. It may seem that the firm should disclose as much "hard" information as possible, to increase the absolute amount of information available to investors and reduce the cost of capital. However, by distorting the relative amounts of hard and soft information, increased disclosure may induce the manager to cut investment, to improve hard information at the expense of soft. Thus, even if the act of disclosure is costless, a high-disclosure policy can be costly. Investment therefore depends on asset pricing variables such as investors' liquidity shocks; disclosure depends (non-monotonically) on corporate finance variables such as growth opportunities and the manager's horizon. Even if a low disclosure policy is optimal to induce investment, the manager may be unable to commit to it. Government intervention to cap disclosure can create value, in contrast to common calls to increase disclosure.

KEYWORDS: Disclosure, managerial myopia, investment, financial and real efficiency, cost of capital.

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An extensive literature analyzes the optimal disclosure policy for a firm, and identifies numerous benefits of greater disclosure. Diamond (1985) shows that disclosing information reduces the need for each individual shareholder to bear the cost of gathering it. In Diamond and Verrecchia (1991), disclosure reduces the cost of capital by lowering the information asymmetry that shareholders suffer if they subsequently need to sell due to a liquidity shock. Kanodia (1980) and Fishman and Hagerty (1989) show that disclosure increases price efficiency and thus the manager's investment incentives.

These theories only apply to the disclosure of "hard" (i.e., quantitative and verifiable) information, as only hard information can be credibly disclosed. "Soft" information cannot be, because it is non-verifiable.[1] For example, a firm can credibly communicate its earnings, but not the quality of its intangible assets such as its human capital, corporate culture, or R&D capability. Zingales (2000) argues that such intangible assets are particularly important in the modern firm.

It may seem that this distinction does not matter: the disclosure of soft information is moot and so firms should simply apply the insights of disclosure theories to hard information. Thus, if the benefits of disclosure outweigh the costs, firms should disclose as much hard information as possible, to minimize the cost of capital (Diamond and Verrecchia (1991)).[2] Indeed, recent government policies have increased disclosure requirements, such as Sarbanes-Oxley, Regulation FD, and Dodd-Frank.

This article reaches a different conclusion. It shows that the insights of disclosure theories cannot simply be applied to hard information alone, if soft information is important. In particular, even if the actual act of disclosing hard information is costless, a high-disclosure policy can be costly. While increased disclosure of hard information increases the *absolute* amount of information, reducing the cost of capital, it also distorts the *relative* amount of hard versus soft information, since the latter cannot be disclosed. In turn, this distorts the manager's investment decision, as it depends on the relative amounts of hard and soft information in the market. If neither type of

[1] See, e.g., Stein (2002) and Petersen (2004) for the distinction between hard and soft information.

[2] The costs of disclosure previously identified by the literature are believed to be less important nowadays. First, the cost of communicating information has dramatically decreased due to electronic communication. Second, the cost of producing information is low since firms already produce copious information for internal and tax purposes. Third, the information may be proprietary (e.g., Verrecchia (1983) and Dye (1986)). While likely important for some types of disclosure (e.g., the stage of a patent application), proprietary considerations are unlikely to be for others (e.g., earnings). Fourth, Hirshleifer (1971) shows that disclosure in insurance markets may worsen risk-sharing, e.g. if it is made public which individuals will suffer heart attacks before they have a chance to take out medical insurance. However, Diamond (1985) argues that this cost is unlikely to be significant for financial markets, where continuous trading is possible.

information is disclosed, the manager invests optimally. Increasing the disclosure of hard information, relative to soft, distorts the manager's actions towards improving the hard signal at the expense of the soft signal – for example, cutting investment in intangible assets to increase current earnings.

Our model features a firm initially owned and run by a manager, who must raise funds from an outside investor. After funds are raised, the firm turns out to be either high or low quality, and this type is unknown to the investor. As in Diamond and Verrecchia (1991), the investor may subsequently suffer a liquidity shock which forces her to trade additional shares. Also present in the market is a speculator (such as a hedge fund) who has private information on firm value, and a market maker. The investor expects to lose to the speculator from her liquidity trading and thus demands a larger stake when contributing funds, augmenting the cost of capital.

The manager can reduce the investor's informational disadvantage, and thus the cost of capital, by disclosing hard information (such as earnings) that is partially informative about firm value, just before the trading stage. We initially assume that the manager can commit to a disclosure policy when raising funds, as in the literature on mandatory disclosure. High disclosure indeed reduces the cost of capital, but has an important cost. A high-quality firm has the option to undertake an intangible investment that improves the firm's long-run value, but also raises the probability of delivering low earnings. If low earnings are disclosed, the firm's stock price falls since a low-quality firm always delivers low earnings. The manager's objective function places weight on both the short-term stock price and long-term firm value.

In a benchmark model in which firm value is hard information, the optimal policy is to maximize disclosure of long-run value. Such a policy reduces the cost of capital and causes no investment distortion, since investment improves both firm value and the disclosed signal (and thus the stock price). The more realistic case is when long-run value is soft information, since it is not realized until the future. Since investment improves soft information but worsens hard information, disclosure induces underinvestment. While existing literature on optimal disclosure typically assumes that firm value is exogenous, here firm value is endogenous to the disclosure policy (even absent a competitor who can use the disclosed information). In contrast to some existing theories, here the cost of disclosure is its effect on real variables – it reduces investment.

The optimal level of disclosure is a trade-off between the benefits of disclosure (reduced cost of capital) and its costs (inefficient investment). Thus, the model predicts how disclosure should vary across firms. Intuition might suggest that firms with better growth opportunities will disclose less, since investment dominates the trade-off, but we show that the effect of growth opportunities is non-monotonic. Up to a point, increases in growth opportunities indeed reduce disclosure: investment becomes sufficiently important that the firm is willing to sacrifice disclosure to pursue it. For example, at the time of its IPO, Google announced that it would not provide earnings guidance as such disclosure would induce short-termism. Their founders' letter stated "we believe that artificially creating short term target numbers serves our shareholders poorly."[3] However, when investment opportunities are very strong, the manager will exploit them fully even when disclosure is high. Thus, disclosure is lowest for firms with intermediate growth opportunities, and high for firms with weak or strong growth opportunities. For similar reasons, disclosure is high either when uncertainty (the difference in value between high- and low-quality firms), shareholders' liquidity shocks, or signal imprecision (the risk that investment leads to a bad signal), are low, as the manager will invest fully even with high disclosure, or when these parameters are high, as disclosure becomes important relative to investment. Surprisingly, an increase in signal imprecision need not induce less disclosure of the signal. Such an increase makes the cost of capital more important relative to investment, and so the manager may choose full disclosure to minimize the cost of capital.

More broadly, by combining investment, disclosure, informed trading, and capital raising within a unifying framework, we generate new empirical predictions linking investment (a corporate finance topic) to informed trading and the cost of capital (asset pricing topics) since both are linked through disclosure. While researchers typically study how investment depends on Tobin's Q or financial constraints, we show that it depends on microstructure features such as shareholders' liquidity needs, since they influence disclosure policy and thus investment. While the cost of capital depends on microstructure features such as information asymmetry, we show that it also depends on corporate finance variables such as growth opportunities and the manager's short-term concerns, since these influence disclosure policy and thus the cost of capital.

We next consider the case in which the manager cannot commit to a disclosure policy, as in the literature on voluntary disclosure. If investment is important, the manager would like to announce a low disclosure policy. However, if he invests and gets lucky, i.e., still delivers high earnings, he will renege on the policy and disclose the high earnings anyway. Then, if the market receives no disclosure, it rationally infers low earnings, else the manager would have released them – the "unraveling" result

[3] Similarly, Porsche was expelled from the M-DAX index in August 2001, after refusing to comply with its requirement for quarterly reporting, arguing that such disclosures would lead to myopia.

of Grossman (1981) and Milgrom (1981). The only dynamically consistent policy is full disclosure, and investment suffers. In this case, government intervention can be desirable. By capping disclosure (for example by increasing verification requirements), it can allow the firm to implement the optimal policy. This conclusion contrasts earlier research which argues that regulation should increase disclosure due to externalities (Foster (1979), Coffee (1984), Dye (1990), Admati and Pfleiderer (2000), and Lambert, Leuz, and Verrecchia (2007)). Our model thus implies that regulations to increase disclosure (e.g., Sarbanes-Oxley) may have real costs.

However, the effect of government intervention on firm value is unclear. First, even if the government's objective function were to maximize initial firm value, the optimal disclosure policy is firm-specific, whereas regulation cannot be tailored to an individual firm. Second, the government's policy may be to maximize total surplus, in which case it ignores investor losses from liquidity shocks, since they are offset by trading profits to the speculator. Then, the government will choose the disclosure policy that maximizes investment, which is inefficiently low for the firm as it leads to a high cost of capital. Third, Regulation FD attempts to "level the playing field" between different investors, suggesting an objective to minimize trading losses for retail investors. In this case, the government will maximize disclosure, at the expense of investment.

This paper is related to a large literature on the costs and benefits of disclosure, which is reviewed by Verrecchia (2001), Dye (2001), Beyer, Cohen, Lys, and Walther (2010), and Goldstein and Sapra (2012). Our main innovation is to show that the existence of soft information is not irrelevant for disclosure policy, as one might suspect because the disclosure of soft information is moot – instead, it introduces a real cost of disclosing hard information. Gigler, Kanodia, Sapra, and Venugopalan (2013) show that an interim signal can induce the manager to choose a short-term project over a long-term alternative, in a setting where both projects are ex ante unprofitable (in contrast to our model). They compare a social planner's payoff across two discrete regimes (with and without the interim signal), assuming that commitment is possible. We study the firm's optimal choice from a continuum of disclosure policies and the interaction with the cost of capital, thus delivering predictions on how firms' disclosure decisions depend (non-monotonically) on asset pricing and corporate finance factors. We also consider the voluntary disclosure case where the firm cannot commit to a disclosure policy. Han, Liu, Tang, Yang, and Yu (2013) show that disclosure can reduce investment through the different channel of attracting noise traders which reduce price informativeness, and thus the manager's ability to learn from the stock price. In

Hermalin and Weisbach (2012), disclosure affects the manager's incentives to engage in manipulation. He prefers less disclosure ex post; here, the manager discloses too much where disclosure is voluntary. Einhorn and Ziv (2007) study a multi-tasking framework and show that firms allocate more resources to divisions whose profitability has a greater effect on the signal of aggregate firm performance. In contrast to our paper, there is no disclosure decision before the resource allocation decision.

Consistent with our theory, survey results by Graham, Harvey, and Rajgopal (2005) suggest that 78% of executives would sacrifice long-term value to meet earnings targets. Bhojraj and Libby (2005) show experimentally that the expectation of future equity sales induces myopia, Cheng, Subrahmanyam, and Zhang (2007) document that firms that issue quarterly earnings guidance invest less in R&D, and Ernstberger, Link, and Vogler (2011) find that European Union firms in countries with quarterly rather than semi-annual reporting engage in greater short-termism. Turning to the benefits of disclosure, Balakrishnan, Billings, Kelly, and Ljungqvist (2013) use a natural experiment to show that greater disclosure increases liquidity and thus reduces the cost of capital. This result is consistent with our model and also with Diamond and Verrecchia (1991).

Other researchers have noted that regulation should sometimes constrain disclosure. Fishman and Hagerty (1990) advocate limiting the set of signals from which the firm may disclose; here the constraint is on the level of disclosure. In Fishman and Hagerty (1989), traders can only acquire a signal in one firm, and so disclosure draws traders away from one's rivals. Here, disclosure is excessive due to a commitment problem, rather than a negative externality. In models where disclosure is a costly signal with no real effects (e.g., Jovanovic (1982), Verrecchia (1983)), disclosure is a deadweight loss. Here, disclosure is costly even though the act of disclosure is costless.

This paper also contributes to a literature on the real effects of financial markets. The survey of Bond, Edmans, and Goldstein (2012) identifies two channels through which financial markets (and thus disclosure) can affect the real economy. Our mechanism operates through the contracting channel: the manager's contract is contingent upon the stock price, and so his incentives to take real decisions depend on the extent to which they will be incorporated in the price. The second channel is that the manager uses information in the stock price to guide his decisions. This mechanism allows for a quite different real cost of disclosure. Disclosing information may reduce speculators' incentives to acquire private information (Gao and Liang (2013)) or to trade aggressively on private information (Bond and Goldstein (2012)). This in turn reduces the information in prices from which the manager can learn.[4]

This literature typically concludes that financial efficiency is desirable for real efficiency.[5] We show that real efficiency is non-monotonic in financial efficiency. The manager invests efficiently if neither (hard) earnings nor (soft) fundamental value are disclosed (in which case financial efficiency is minimized), and also if both are disclosed (in which case financial efficiency is maximized). When soft information cannot be disclosed, then even though disclosure of hard information augments financial efficiency, it reduces real efficiency. It may be better for prices to contain no information than partial information. This result echoes the theory of the second best, where it may be optimal to tax all goods rather than a subset. Holmstrom and Milgrom (1991) show that difficulties in measuring one task may lead to the principal optimally offering weak incentives for all tasks. Our result also echoes Paul (1992), who shows that an efficient financial market weights information according to its informativeness about asset value, but to incentivize efficient real decisions, information should be weighted according to its informativeness about the manager's actions. While a higher hard signal is a positive indicator of firm type, it is a negative indicator of investment.

This paper is organized as follows. Section 1 lays out the model. Section 2 analyzes the case in which the firm can commit to disclosure and solves for the optimal policy. Section 3 considers the case of voluntary disclosure and introduces a role for regulation, and Section 4 concludes. Appendix A contains all proofs not in the main text.

1 The Model

The model consists of four players. The *manager* initially owns the entire firm and chooses its disclosure and investment policies. The *investor* contributes equity financing and may subsequently suffer a liquidity shock. The *speculator* has private information on firm value and trades on this information. The *market maker* clears the market and sets prices. All players are risk-neutral and there is no discounting.

[4] Other costs of disclosure need not operate through the real effects of financial markets. In Morris and Shin (2002), an agent's optimal decision depends on his expectation of other agents' actions (e.g. whether to run on a bank, or whether to buy a product with network externalities). The agent rationally over-reacts to publicly disclosed information, since he takes into account other agents' reactions to the information, and so under-utilizes his own private information. In Pagano and Volpin (2012) and Di Maggio and Pagano (2012), disclosed information can be understood costlessly by speculators but not by hedgers, and so disclosure increases information asymmetry.

[5] In these models, the price is always semi-strong-form "efficient", regardless of disclosure, in that it equals expected firm value conditional upon an information set. Greater disclosure means that the price is now efficient relative to a richer information set. We refer to this as greater price efficiency.

There are five periods. At $t = 0$, the manager must raise financing of K, which is injected into the firm. He first commits to a disclosure policy $\sigma \in [0, 1]$ and then sells a stake α to the investor, which is chosen so that the investor breaks even.

The firm has two possible types, $\theta \in \Theta \equiv \{L, H\}$, that occur with equal probability. Type L (H) corresponds to a low- (high-) quality firm. At $t = 1$, the firm's type θ is realized. We will sometimes refer to a firm of type θ as a "θ-firm" and its manager as a "θ-manager". As in the myopia model of Edmans (2009), an L-manager has no investment decision and his firm will be worth $V^L = R^L$ at $t = 4$, but an H-manager chooses an investment level $\lambda \in [0, 1]$ and his firm is worth $R^H + \lambda g$ at $t = 4$, where $g > 0$ parameterizes the desirability of the investment opportunity.[6] (All values are inclusive of the K raised by the financing.) Since $g > 0$, $\lambda = 1$ is first-best. The type θ and the investment level λ are observable to both the manager and the speculator (and so both know V), but neither are observable to the investor and market maker.

At $t = 2$, a hard (verifiable) signal $y \equiv \{G, B, \varnothing\}$ (such as earnings) is generated. With probability $1 - \sigma$, the signal is the null signal $\varnothing$, which corresponds to no disclosure. With probability σ, a partially informative signal is disclosed. An L-firm always generates signal B. An H-firm generates B with probability $\rho\lambda^2$ and G with probability $1 - \rho\lambda^2$. The variable $\rho \in (0, 1)$ parameterizes the extent to which investment increases the probability of $y = B$; we will sometimes refer to ρ as the noise in the signal.

At $t = 3$, the investor suffers a liquidity shock with probability ϕ, which forces her to either buy or sell β shares with equal probability. With probability $1 - \phi$, she suffers no shock; she will not trade voluntarily as she is uninformed. Her trade is therefore given by $I = \{-\beta, 0, \beta\}$. If $y = G$, the public signal is fully informative and so the speculator will not trade, but if $y \in \{B, \varnothing\}$, the public signal is not fully informative and the speculator will take advantage of his private information on V by trading an amount S. Similar to Dow and Gorton (1997), the market maker observes each individual trade, but not the identity of each trader. For example, if the vector of trades Q equals $(-\beta, \beta)$, he does not know which trader (speculator or investor) bought β, and which trader sold β. The market maker is competitive and sets a price P equal to expected firm value conditional upon the observed trades. He clears any excess demand or supply from his own inventory.

[6]The specification $V^H = R^H + \lambda g$ implies that the growth opportunity is independent of the amount of financing raised (e.g. the funds K could be required to repay debt, rather than to fund the growth opportunity). The model's results remain unchanged to parameterizing $g = hK$, so that the growth opportunity does depend on the amount of financing raised.

At $t = 4$, firm value $V \in \{V^H, V^L\}$ becomes known and payoffs are realized. We consider two versions of the model. In a preliminary benchmark, V is hard information and can be credibly disclosed at $t = 2$. In the core model, V is soft information prior to $t = 4$ and thus cannot be credibly disclosed.[7] Note that soft information is still present in the model, because the speculator has information on V and trades on it.

The manager's objective function is $(1 - \alpha)(\omega P + (1 - \omega) V)$. After raising financing, the manager's stake in the firm is $(1 - \alpha)$. The concern for the short-term stock price $\omega \in (0, 1)$ is standard in the myopia literature and can arise from a number of sources introduced by prior research: takeover threat (Stein (1988)), concern for managerial reputation (Narayanan (1985), Scharfstein and Stein (1990))[8], or the manager expecting to sell a fraction ω of his remaining shares just after $t = 3$ and hold the remaining $1 - \omega$ until $t = 4$, as in Stein (1989).

Before solving the model, we discuss its assumptions. Investment improves fundamental value but potentially lowers earnings, as in the classic myopia models of Stein (1988, 1989). Investment in R&D, advertising, or training employees is nearly always expensed; investors cannot distinguish whether high expenses are due to desirable investment (an H-firm choosing a high λ) or low firm quality (an L-firm). Similarly, even though R&D and advertising can be separated out in an income statement, outside investors do not know whether high R&D or advertising is efficient, or stems from a low-quality manager wasting cash. Also as in myopia models, short-term earnings are verifiable but long-run fundamental value is not (prior to the final period) in the core model. Intangible investment does not pay off until the long run, and it is very difficult for the manager to credibly certify the quality of his firm's intangible assets (e.g., its corporate culture).

Outside investors have no information on the firm's type, and the speculator has perfect information. This seemingly stark dichotomy is purely for simplicity; we only require the speculator to have some information advantage over outside investors. Many shareholders (e.g., retail investors) are atomistic and lack the incentive to gather information about the firm, or are unsophisticated and lack the expertise to do so. Speculators such as hedge funds often closely monitor firms that they do not currently have a stake in to generate trading ideas.

[7] In Almazan, Banerji, and De Motta (2008), the signal is soft but disclosure matters because it may induce a speculator to investigate the disclosure. Here, any disclosure of V is non-verifiable.

[8] Under these interpretations, it may seem that a more natural objective function is $(1 - \alpha) V + \xi P$ where $(1 - \alpha) V$ is the value of the manager's stake and ξP represents his short-term concerns from these additional sources. The objective function of $(1 - \alpha)(\omega P + (1 - \omega) V)$ is simply $1 - \omega$ times this objective function, where $\xi = \frac{(1-\alpha)\omega}{1-\omega}$.

The liquidity-enforced selling occurs because the investor may suffer a sudden demand for funds, e.g., to pursue another investment opportunity. Liquidity-enforced buying occurs because the investor may have a sudden inflow of cash. She will invest a disproportionate fraction of these new funds into the firm if she is less aware of stocks she does not currently own (e.g., Merton (1987)).[9] The results continue to hold if the investor only faces the probability of liquidity-enforced selling. All we require is that the investor may have to trade against a more informed speculator, regardless of the direction of her trade, as in Diamond and Verrecchia (1991).

We now formally define a Perfect Bayesian Equilibrium as our solution concept.

Definition 1 *The manager's disclosure policy* $\sigma \in [0,1]$*, the* H*-manager's investment strategy* $\lambda : [0,1] \to [0,1]$*, the speculator's trading strategy* $S : \Theta \times [0,1] \times \{G, B, \varnothing\} \to \mathbb{R}$*, the market maker's pricing strategy* $P : [0,1] \times \{G, B, \varnothing\} \times \mathbb{R}^2 \to \mathbb{R}$*, the market maker's belief* μ *about* $\theta = H$*, and the belief* $\hat{\lambda}$ *about the* H*-manager's investment level constitute a Perfect Bayesian Equilibrium, if:*

1. *given* μ *and* $\hat{\lambda}$*,* P *causes the market maker to break even for any* $\sigma \in [0,1]$*,* $y \in \{G, B, \varnothing\}$*, and* $Q \in \mathbb{R}^2$*;*

2. *given* $\hat{\lambda}$ *and* P*,* S *maximizes the speculator's payoff for any* V*,* $\sigma \in [0,1]$*, and* $y \in \{G, B, \varnothing\}$*;*

3. *given* S *and* P*,* λ *maximizes the* H*-manager's payoff given* $\sigma \in [0,1]$*;*

4. *given* λ*,* S*, and* P*,* σ *maximizes the manager's payoff;*

5. *the belief* μ *is consistent with the strategy profile; and*

6. *the belief* $\hat{\lambda} = \lambda$*, i.e., is correct in equilibrium.*

2 Analysis

2.1 First-Best Benchmark

As a benchmark, we first assume that V is hard information, i.e., the manager can commit to disclosing it with probability σ_V. If V is disclosed, then $P = V$ regardless of the order flow. Thus, the investor makes no trading losses and the H-manager faces

[9] In Holmstrom and Tirole (1993), Bolton and von Thadden (1998), Kahn and Winton (1998), and Edmans (2009), liquidity purchases also stem from existing owners.

no trade-off between stock price and fundamental value when investing. He chooses $\lambda = 1$ as this maximizes both.

Since disclosure of V both maximizes investment and minimizes the cost of capital, the manager chooses $\sigma_V = 1$. Thus, financial and real efficiency are both maximized and the first best is achieved. Since y is uninformative conditional upon V, the manager's disclosure policy σ for the signal y is irrelevant, and so he is indifferent between any $\sigma \in [0, 1]$. This result is given in Lemma 1 below.

Lemma 1 *(Disclosure of fundamental value): If fundamental value V is hard information, the manager chooses $\sigma_V = 1$, $\lambda^* = 1$, and any $\sigma \in [0, 1]$.*

We now turn to the core model in which V is soft information and thus cannot be disclosed. We solve this model by backward induction. We start by determining the stock price at $t = 3$, given the market's belief about the manager's investment. We then move to the manager's $t = 2$ investment decision, which is a best response to the market maker's $t = 3$ pricing function. Finally, we turn to the manager's choice of disclosure at $t = 0$, which takes into account the impact on his subsequent investment decision and the investor's losses from liquidity shocks.

2.2 Trading Stage

The trading game at $t = 3$ is played by the speculator and the market maker. At this stage, the manager's investment decision λ (if $\theta = H$) has been undertaken, but is unknown to the market maker. Thus, he sets the price using his equilibrium belief $\hat{\lambda}$.

There are three cases to consider. If $y = G$, all players know that $\theta = H$, so the unique equilibrium in this subgame is that the market maker sets $P = \widehat{V^H} = R^H + \hat{\lambda} g$. Since the speculator values the firm at V^H (and, in equilibrium, $\hat{\lambda} = \lambda$), he has no motive to trade. If the investor suffers a liquidity shock, she trades at a price of $P = \widehat{V^H}$ and breaks even. When $y = B$, the signal is imperfectly informative for any $\hat{\lambda} > 0$: it can be generated by both types. Since the speculator observes V, he has an information advantage. Because the investor either buys or sells β shares (or does not trade), the speculator will buy β shares if $V = V^H$ and sell β shares if $V = V^L$, to hide his information. Similarly, when $y = \varnothing$, the speculator has an information advantage and again will trade.

Given the speculator's equilibrium strategy, the market maker's equilibrium pricing function is given by Bayes' rule in Lemma 2.

Lemma 2 *(Prices): Upon observing signal y and the vector of order flows Q, the prices set by the market maker are given by the following table:*

Q	(β, β)	$(\beta, 0)$	$(\beta, -\beta)$	$(-\beta, 0)$	$(-\beta, -\beta)$
$P(y = \varnothing)$	$\widehat{V^H}$	$\widehat{V^H}$	$\frac{1}{2}\left(\widehat{V^H} + V^L\right)$	V^L	V^L
$P(y = B)$	$\widehat{V^H}$	$\widehat{V^H}$	$\frac{\rho\hat{\lambda}^2}{1+\rho\hat{\lambda}^2}\widehat{V^H} + \frac{1}{1+\rho\hat{\lambda}^2}V^L$	V^L	V^L
$P(y = G)$	$\widehat{V^H}$				

(1)

We use $P(Q, y)$ to denote the price of a firm for which signal y has been disclosed and the order vector is Q. Since y is an informative signal, financial efficiency is greater with $y = B$ than $y = \varnothing$. This can be seen by the difference in prices with an uninformative order vector of $(\beta, -\beta)$. Without a signal, the price is the unconditional expected value based on the prior probability of type H $(\frac{1}{2})$, but conditional on $y = B$, the probability is updated to the posterior $\frac{\rho\hat{\lambda}^2}{\rho\hat{\lambda}^2+1}$. Separately, it is simple to show that, at $t = 2$, $\mathbb{E}(P) = \mathbb{E}(V)$ – a consequence of market efficiency.

Let $\widetilde{P}(y|\theta = H)$ denote the expected stock price of an H-firm for which signal y has been disclosed, where the expectation is taken over the possible realizations of order flow. We thus have:

$$P(G|\theta = H) = \widehat{V^H},$$

$$\tilde{P}(B|\theta = H) = \widehat{V^H} - \frac{\phi}{2}\frac{\widehat{V^H} - V^L}{1 + \rho\hat{\lambda}^2}, \text{ and}$$

$$\tilde{P}(\varnothing|\theta = H) = \widehat{V^H} - \frac{\phi}{2}\frac{\widehat{V^H} - V^L}{2},$$

where we suppress the tilde on $P(G|\theta = H)$ as the price is independent of the order flow. For any σ and $\hat{\lambda}$, since $\widehat{V^H} > V^L$ and $\rho\hat{\lambda}^2 < 1$, we have

$$\tilde{P}(B|\theta = H) < \tilde{P}(\varnothing|\theta = H) < P(G|\theta = H).$$

2.3 Investment Stage

We now move to the investment decision of the H-manager at $t = 2$. At this stage, the disclosure policy σ is known. The manager chooses λ to maximize his expected payoff:

$$\max_{\lambda} U_m\left(\lambda, \widehat{\lambda}\right) = (1 - \alpha)\left(\omega\mathbb{E}(P|\theta = H) + (1 - \omega)V^H\right), \tag{2}$$

where the expected price of an H-firm is

$$\begin{aligned}\mathbb{E}(P|\theta = H) &= \sigma\left(1-\rho\lambda^2\right)P(G|\theta = H) + \sigma\rho\lambda^2\tilde{P}(B|\theta = H) \\ &\quad + (1-\sigma)\tilde{P}(\varnothing|\theta = H) \\ &= \widehat{V^H} - \frac{\phi}{2}\left(\frac{1}{2}(1-\sigma) + \sigma\frac{\rho\lambda^2}{1+\rho\hat{\lambda}^2}\right)\left(\widehat{V^H} - V^L\right).\end{aligned}$$

His first-order condition is given by

$$\frac{\partial U_m\left(\lambda,\widehat{\lambda}\right)}{\partial\lambda} = (1-\alpha)\left(-\omega\phi\sigma\frac{\rho\lambda}{1+\rho\hat{\lambda}^2}\left(\widehat{V^H} - V^L\right) + (1-\omega)g\right) = 0. \tag{3}$$

Since $\frac{\partial^2 U_m\left(\lambda,\widehat{\lambda}\right)}{\partial\lambda^2} < 0$, the manager's objective function is strictly concave and so equation (3) is sufficient for a maximum. Plugging $\lambda = \widehat{\lambda}$ into (3) yields the quadratic:

$$\Psi(\lambda,\sigma) = \left(\frac{1}{\Omega} - \sigma\phi\right)\lambda^2 - \sigma\phi\frac{\Delta}{g}\lambda + \frac{1}{\Omega\rho}, \tag{4}$$

where we define $\Omega \equiv \frac{\omega}{1-\omega}$ as the relative weight on the stock price and $\Delta \equiv R^H - R^L$ as the difference in firm values.

Given a σ, the manager's investment decision is given in Proposition 1 below.

Proposition 1 *(Investment): For any $\sigma \in [0,1]$, there is a unique equilibrium investment level in the subgame following σ, which is given by:*

$$\lambda^* = \begin{cases} r(\sigma), & \text{if} \quad \sigma > X; \\ 1, & \text{if} \quad \sigma \leq X, \end{cases}$$

where

$$X \equiv \frac{g(\rho+1)}{\Omega\phi\rho(\Delta+g)}, \tag{5}$$

$r(\sigma)$ is the root of the quadratic $\Psi(\lambda,\sigma) = 0$ for which $\Psi'(r,\sigma) < 0$. It is strictly decreasing and strictly convex. Fixing any $\sigma > X$, the partial investment level $r(\sigma)$ is increasing in g and decreasing in ω, ϕ, ρ, and Δ. The threshold X is increasing in g and decreasing in ω, ϕ, ρ, and Δ.

The intuition behind Proposition 1 is as follows. The cost of investment (from the manager's perspective) is that it increases the probability of disclosing a bad signal.

This cost is increasing in disclosure σ. Thus, the manager engages in full investment if and only if σ is sufficiently low. As is intuitive, $\sigma \leq X$ is more likely to be satisfied if ω is low (the manager is less concerned with the stock price), ρ is low (investment only leads to a small increase in the probability of a bad signal) and g is high (investment is more attractive). Somewhat less obviously, $\sigma \leq X$ is more likely to be satisfied if ϕ is low. When the investor receives fewer liquidity shocks, trading becomes dominated by the speculator, who has information on V. The price becomes more reflective of V rather than y. Thus, the manager is less concerned about emitting the bad signal. Finally, investment is likelier if Δ, the baseline value difference between H- and L-firms, is low, as this reduces the incentive to be revealed as H by delivering $y = G$.

When $\sigma > X$, disclosure is sufficiently frequent that the manager reduces investment below the first-best optimum, and we have an interior solution. Additional increases in σ cause investment to fall further, since $r(\sigma)$ is decreasing in σ. Thus, while a rise in σ augments financial efficiency, it reduces real efficiency.

2.4 Disclosure Stage

We finally turn to the manager's disclosure decision at $t = 0$. He chooses σ to maximize his expected payoff, net of the stake sold to outside investors:

$$\begin{aligned}\max_{\sigma} \Pi(\sigma) &= (1 - \alpha(\sigma)) \left(\omega \mathbb{E}(P) + (1-\omega)\mathbb{E}[V]\right) \\ &= (1 - \alpha(\sigma)) \, \mathbb{E}[V]. \end{aligned} \tag{6}$$

The manager takes into account two effects of σ. First, it affects α, because the investor's stake must be sufficient to compensate for her trading losses. Second, it affects λ and thus V^H, as shown in Proposition 1. Lemma 3 addresses the first effect.[10]

Lemma 3 *(Stake sold to investor): The stake α sold to the investor is given by*

$$\alpha(\sigma) = \frac{2K}{V^H + R^L} + \kappa, \tag{7}$$

where

$$\kappa = \frac{\beta\phi\left(V^H - R^L\right)\left[\frac{1}{2}(1-\sigma) + \frac{\rho\lambda^2}{1+\rho\lambda^2}\sigma\right]}{V^H + R^L}. \tag{8}$$

[10] The stake demanded by the investor depends on her conjecture for the manager's investment decision, $\widehat{\lambda}$. In equilibrium, $\widehat{\lambda}$ =sλ, and so λ appears in Lemma 3.

The partial derivative of κ with respect to σ is negative, and the partial derivatives with respect to ω, ϕ, ρ, β, λ, and g are positive.

Lemma 3 shows that the stake α comprises two components. The "baseline" component $\frac{2K}{V^H+R^L}$ is the stake that the investor would require if she did not risk trading losses (e.g., if $\phi = 0$). It is her investment K divided by expected firm value, and independent of σ. The second term κ is the additional stake that she demands to compensate for her expected trading losses. An increase in σ reduces these losses and thus α. We will refer κ as the "excess cost of capital" (or "cost of capital" for short).

The partial derivatives for κ are intuitive. An increase in the noise in the public signal ρ raises the investor's information disadvantage. The probability ϕ and magnitude β of a liquidity shock also increases the expected loss and thus the cost of capital. Disclosure σ reduces information asymmetry and thus the cost of capital. Increases in investment λ and the productivity of investment g both augment the value difference between H- and L-firms $(\Delta + \lambda g)$ and thus the cost of capital.

Plugging (7) into (6) yields

$$\Pi(\sigma) = \left[\frac{1}{2}\left(V^H + R^L\right) - K\right] - \beta\phi\frac{1}{2}\left(V^H - R^L\right)\left[\frac{1}{2}(1-\sigma) + \frac{\rho\lambda^2}{1+\rho\lambda^2}\sigma\right],$$

where the first term is expected firm value (net of the injected funds) and the second term represents the investor's expected trading losses.

We now solve for the manager's choice of disclosure policy. There are two cases to consider. The first is $X \geq 1$. Since $\sigma \in [0,1]$, $\sigma \leq X$. From Proposition 1, we have $\lambda^* = 1 \; \forall \; \sigma$. Since there is no trade-off between disclosure and investment, the manager chooses maximum disclosure, $\sigma^* = 1$. Thus, full disclosure and full investment can be implemented simultaneously. This result is stated in Proposition 2.

Proposition 2 *(Full disclosure and full investment): If $X \geq 1$, the model has a unique equilibrium, in which the disclosure policy is $\sigma^* = 1$ and the investment level is $\lambda^* = 1$.*

The condition $X \geq 1$ is equivalent to

$$\phi\frac{\rho}{1+\rho}\frac{\Delta+g}{g}\Omega \leq 1. \tag{9}$$

The manager will invest efficiently even with full disclosure when g is high, and ω, ϕ, ρ, and Δ are all low. The intuition is the same as in the discussion of Proposition 1.

The second case is $X < 1$. In this case, we solve for the manager's choice of disclosure policy in two steps. First, we solve for the optimal disclosure policy in the set $[0, X]$ (i.e., if the manager implements full investment), and then in $[X, 1]$ (i.e., if the manager implements partial investment).[11] Second, we solve for the optimal disclosure policy overall, which involves comparing the manager's payoffs under the best outcome in $[0, X]$ with full investment, to the best outcome in $[X, 1]$ with partial investment.

We first analyze optimal disclosure in $[0, X]$. From Proposition 1, $\lambda^*(\sigma) = 1$ for all $\sigma \in [0, X]$. Since full investment arises for all σ, the manager chooses the highest σ that supports full investment, which is X. This result is stated in Lemma 4 below:

Lemma 4 *(Disclosure under full investment): In an equilibrium where $\sigma \in [0, X]$ and $X < 1$, the optimal disclosure policy is*

$$\sigma^* = X,$$

and the equilibrium investment level is $\lambda^* = 1$.

We next turn to optimal disclosure in $[X, 1]$. For any $\sigma \in [X, 1]$, the equilibrium in the following subgame is $r(\sigma)$. Thus, the manager's problem becomes

$$\Pi(\lambda, \sigma) = \left[\frac{1}{2}\left(R^H + \lambda g + R^L\right) - K\right] - \frac{1}{4}\beta\phi(\Delta + \lambda g) + \frac{1}{4}\beta\phi(\Delta + \lambda g)\,\sigma\frac{1 - \rho\lambda^2}{1 + \rho\lambda^2}. \quad (10)$$

From $\Psi(\lambda, \sigma) = 0$, the disclosure policy σ that implements a given investment level λ is given by:

$$\sigma = \frac{g\left(1 + \rho\lambda^2\right)}{\lambda\Omega\phi\rho(\Delta + \lambda g)}. \quad (11)$$

As shown in Proposition 1, $r(\sigma)$ is strictly decreasing and strictly convex. Since $\frac{\partial\lambda}{\partial\sigma} < 0$, this implies that σ is strictly decreasing and strictly convex in λ. Increased disclosure reduces investment; since investment cannot fall below zero, it does so at a decreasing rate.

The first term in equation (10) is expected firm value. The second term represents the investor's losses in the absence of disclosure ("maximum trading losses"). The third term constitutes the reduction in expected losses that stems from increased disclosure ("loss mitigation"). This reduction is increasing in the initial variance in firm value $(\Delta + \lambda g)$ and decreasing in λ due to the signal distortion effect.

[11] Since $r(\sigma)$ is continuous at $\sigma = X$ $(r(X) = 1)$, X lies in both sets. This implies that both sets are compact and thus an optimal disclosure policy exists in each.

Using (11) to substitute for σ in the objective function (10) yields firm value as a function of investment alone:

$$\Pi(\lambda) = D + E\lambda + \frac{F}{\lambda}, \tag{12}$$

where

$$D \equiv R^H - \frac{1}{2}(1 + \frac{1}{2}\beta\phi)\Delta - K, \tag{13}$$

$$E \equiv g\left[1 - \frac{1}{2}(1 + \frac{1}{2}\beta\phi) - \frac{\beta}{4\Omega}\right], \text{ and} \tag{14}$$

$$F \equiv \frac{\beta g}{4\rho\Omega}. \tag{15}$$

The convex component in firm value (the $\frac{1}{\lambda}$ term) comes from substituting σ into the loss mitigation term. Since $\Pi(\lambda)$ is globally convex (due to the convexity of $\frac{F}{\lambda}$), the solution to $\Pi'(\lambda) = 0$ is a minimum. The maximum value of $\Pi(\lambda)$ is attained at a boundary: we have either $\lambda^* = r(X) = 1$ or $\lambda^* = r(1)$. The intuition is as follows. From (10), the benefits of increasing investment are linear in λ. One of the costs, the maximum trading losses, is also linear, but the loss mitigation term is convex, because disclosure is convex in investment as shown by (11). Raising investment requires disclosure to fall, reducing the loss mitigation effect, but this fall is at a decreasing rate. Intuitively, when disclosure is already low, further decreases in disclosure have a large relative effect, and so an increase in investment only requires a small decrease in disclosure. The convexity is likely common to all functional forms: since disclosure and investment are bounded below by zero, an increase in disclosure must reduce investment at a declining rate. Hence, if it is optimal for the manager to increase disclosure from X to $X + \varepsilon$, it is optimal to increase it all the way to 1. Thus, he chooses either full investment or full disclosure. This result is given in Lemma 5 below.

Lemma 5 *(Partial disclosure or partial investment): When $\sigma \in [X, 1]$, the equilibrium investment level is either $\lambda^* = r(1)$, in which case the equilibrium disclosure policy is $\sigma^* = 1$, or $\lambda^* = 1$, in which case the equilibrium disclosure policy is $\sigma^* = X$.*

We now move to the second step. Having found the optimal disclosure policy in $[0, X]$ and in $[X, 1]$, we now solve for the optimal disclosure policy overall, by comparing the manager's payoff across these two sets ($\Pi(r(1), 1)$ versus $\Pi(1, X)$). In doing so, we formally prove existence of an equilibrium in the model and characterize it. The equilibrium is given by Proposition 3 below:

Proposition 3 *(Trade-off between disclosure and investment): If $X < 1$, the equilibrium is given as follows:*

(i) *If $\beta > \widetilde{\beta}$, the manager chooses full disclosure ($\sigma^* = 1$) and partial investment ($\lambda^* = r(1) < 1$).*

(ii) *If $\beta < \widetilde{\beta}$, the manager chooses partial disclosure ($\sigma^* = X$) and full investment ($\lambda^* = 1$).*

(iii) *If $\beta = \widetilde{\beta}$ both ($\lambda^* = r(1)$, $\sigma^* = 1$) and ($\lambda^* = 1$, $\sigma^* = X$) are equilibria.*

The threshold $\tilde{\beta}$ is given by

$$\widetilde{\beta} = \frac{1 - r(1)}{\phi \frac{1}{2} \frac{\Delta + g}{g} - \frac{1}{\Omega} \left[\frac{1}{2} \left(\frac{1}{\rho} - 1 \right) + r(1) \right]} > 0. \tag{16}$$

It increases in g, decreases in ϕ, ρ, and Δ, and is U-shaped in ω.

When $X < 1$, the manager chooses between full disclosure or full investment. He chooses the former if and only if the liquidity shock β is sufficiently large (above a threshold $\widetilde{\beta}$), as then cost of capital considerations dominate the trade-off. Importantly, the partial investment level $r(1)$ is independent of $\widetilde{\beta}$, which is why we use β as the cut-off parameter.

The intuition behind the comparative statics for the threshold $\widetilde{\beta}$ arises because changes in parameters have up to three effects. First, as g rises, and ϕ, ρ, and Δ fall, (5) shows that the maximum disclosure X that implements full investment is higher. Full investment becomes more attractive to the manager, as it can be sustained with a lower cost of capital. Second, the same changes also augment the partial investment level $r(1)$ that is implemented by full disclosure. Thus, full disclosure also becomes more attractive, as it leads to less underinvestment. These two effects work in opposite directions. This ambiguity is resolved through a third effect: a rise in g, and a fall in ϕ, ρ, and Δ, make investment more important relative to the cost of capital. Thus, they augment the cutoff $\widetilde{\beta}$, making it more likely that full investment is optimal.

In contrast, a fall in ω only has the first two effects: it reduces both $r(1)$ and X, making both the full disclosure and full investment equilibria less attractive. Since ω affects neither the value of the growth opportunity nor the cost of capital, the third effect is absent, and so the effect of ω on $\widetilde{\beta}$ is ambiguous. When ω is very low, full investment can be sustained with high disclosure and so the manager prefers the full investment equilibrium. When ω is very high, full disclosure leads to substantial underinvestment and so the manager again prefers the full investment equilibrium. The manager chooses full disclosure for intermediate values of ω, and so the derivative of $\widetilde{\beta}$ with respect to ω is non-monotonic.

We now combine the comparative static analysis of cases of $X < 1$ and $X \geq 1$ to analyze how parameters globally affect equilibrium disclosure and investment. Proposition 4 gives the global comparative statics.

Proposition 4 *(Global comparative statics):*

(i) *Equilibrium investment λ^* is weakly increasing in the profitability of investment g. Equilibrium disclosure σ^* is first weakly decreasing and then weakly increasing in g.*

(ii) *Equilibrium investment λ^* is weakly decreasing in the difference in firm values Δ. Equilibrium disclosure σ^* is first weakly increasing and then weakly decreasing in Δ.*

(iii) *Equilibrium investment λ^* is weakly decreasing in the probability of the liquidity shock ϕ. Equilibrium disclosure σ^* is first weakly decreasing and then weakly increasing in ϕ.*

(iv) *Equilibrium investment λ^* is weakly decreasing in the noise in the signal ρ. Equilibrium disclosure σ^* is first weakly decreasing and then weakly increasing in ρ.*

(v) *Equilibrium investment λ^* is weakly decreasing in the manager's short-term con-*

cerns ω. Equilibrium disclosure σ^* is non-monotonic in ω.

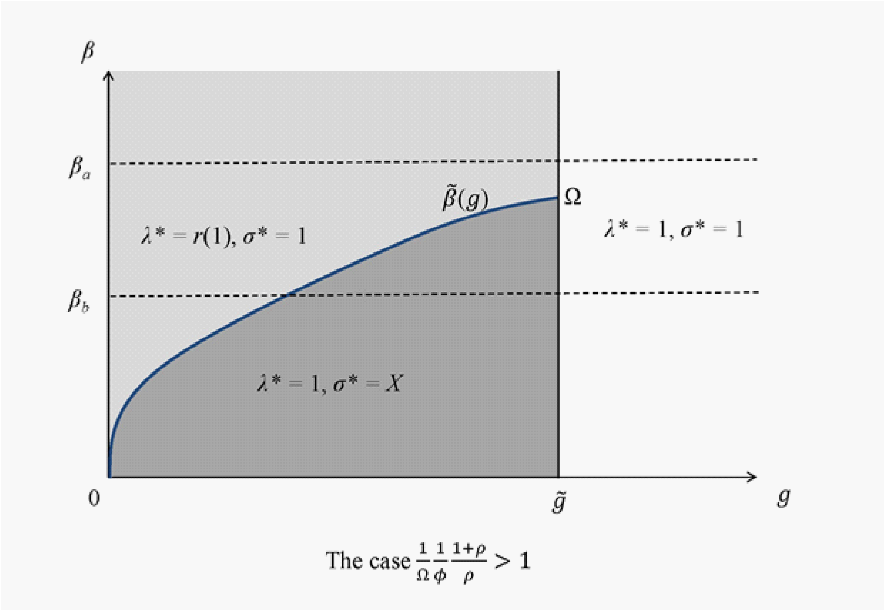

Figure 1: Global comparative statics for g

More precise details on the comparative statics are given in the proof of Proposition 4 in Appendix A. The comparative statics for g depends on whether X is bounded above by 1. The case of $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} > 1$ is illustrated in Figure 1 and the intuition is as follows. There exists $\widetilde{g}$ such that, if $g \geq \widetilde{g}$, then $X \geq 1$ and so we have the $(\lambda^* = 1, \sigma^* = 1)$. For $g < \widetilde{g}$ we have two cases. If $\beta > \Omega$ (e.g., at β_a in Figure 1), the firm chooses partial investment for all $g < \widetilde{g}$. If $\beta < \Omega$ (e.g., at β_b), the firm chooses partial investment only when g is low. Within the partial investment regime, increases in g augment the partial investment level, but do not affect disclosure which remains fixed at 1. When g rises above a threshold (i.e., crosses the solid curve), investment becomes sufficiently attractive that we move to full investment. At the threshold, investment rises discontinuously to 1 and disclosure drops discontinuously from 1 to X. Further increases in g augment disclosure, because the investment opportunity is sufficiently attractive that the manager invests fully even with high disclosure. The case of $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} \leq 1$ (so that $X < 1 \ \forall \ g$) is similar except that we never reach the $(\lambda^* = 1, \sigma^* = 1)$ equilibrium.

Overall, investment is weakly increasing in g. As investment becomes more attractive, the manager pursues it to a greater extent even with full disclosure, and after a point it becomes so attractive that the manager switches to full investment. The effect of g on disclosure is more surprising. Increases in g make investment more important and induce the manager to reduce disclosure, to implement full investment. However, within the full investment equilibrium, further increases in g increase disclosure.

The intuition for Δ is exactly the opposite, because Δ and g appear together as the ratio $\frac{\Delta+g}{g}$ in both X and $\tilde{\beta}$. The manager trades off the benefits of investment g with the incentive to be revealed as a H-firm, Δ.

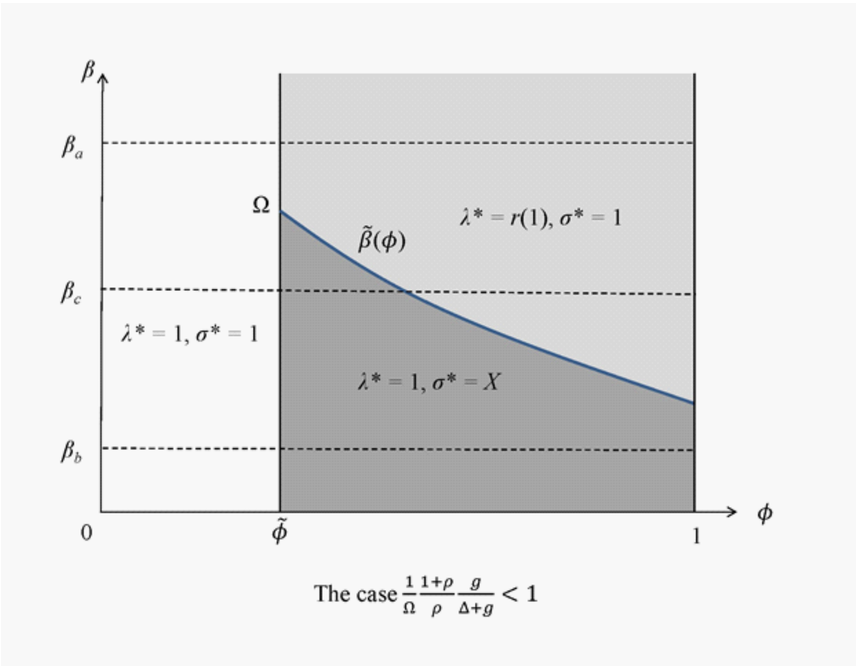

Figure 2: Global comparative statics for ϕ

The intuition behind the global comparative statics for ϕ is as follows. When $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} \geq 1$, then (9) is satisfied for all ϕ. Thus, we always have $X \geq 1$ and the $(\lambda^* = 1,\ \sigma^* = 1)$ equilibrium. The benefits of investment are so strong relative to the costs that, regardless of ϕ, full investment and full disclosure can be sustained simultaneously. Thus, there are no comparative statics with respect to ϕ. The interesting case of $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} < 1$ is illustrated in Figure 2. For low ϕ, $X \geq 1$ and the $(\lambda^* = 1,\ \sigma^* = 1)$ equilibrium is sustainable. When ϕ crosses a threshold $\widetilde{\phi}$, $X < 1$ and $(\lambda^* = 1,\ \sigma^* = 1)$ is no longer sustainable; there is a trade-off between investment and disclosure. We have three cases. When $\beta > \Omega$ (e.g., at β_a in Figure 2), $\beta > \widetilde{\beta}\ \forall\ \phi$. Thus, for $\phi > \widetilde{\phi}$, the manager always chooses partial investment. Investment falls below 1 when ϕ crosses above $\widetilde{\phi}$; additional increases in ϕ reduce the partial investment level further. When β is low (e.g., at β_b), $\beta > \widetilde{\beta}\ \forall\ \phi$. Thus, for $\phi > \widetilde{\phi}$, the manager always chooses partial disclosure. Disclosure falls below 1 when ϕ crosses above $\widetilde{\phi}$; additional increases in ϕ reduce the partial disclosure level further. When β is intermediate (e.g., at β_c), then when $\phi > \widetilde{\phi}$ but remains low, $\beta < \widetilde{\beta}$ and the manager chooses partial disclosure, but for when ϕ crosses the solid curve, $\beta > \widetilde{\beta}$ and the manager switches to partial investment.

Considering all cases together, as with g and Δ in Proposition 4, ϕ has a monotonic effect on investment, but a non-monotonic effect on disclosure. The intuition behind the global comparative statics for ρ is identical.

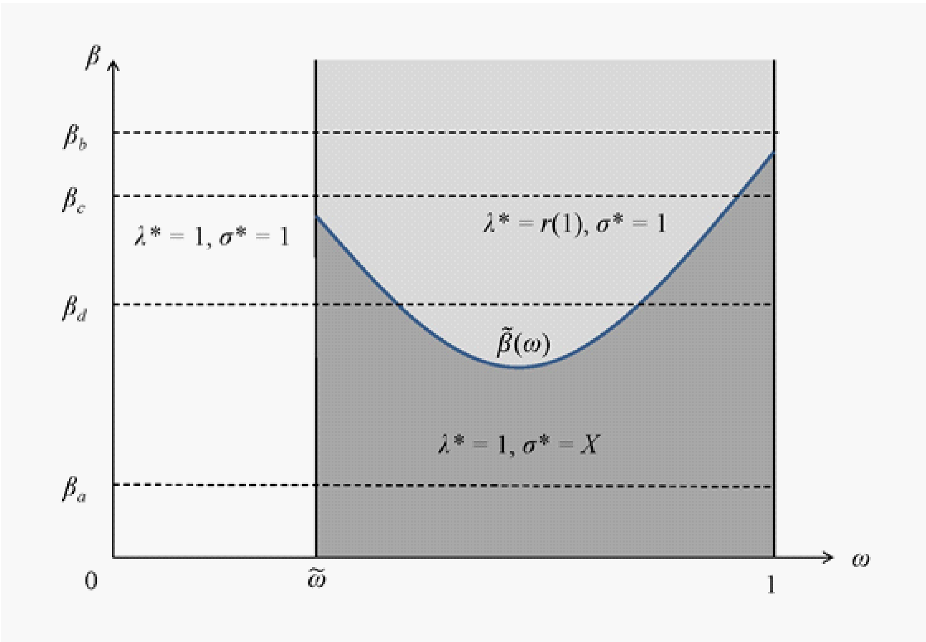

Figure 3: Global comparative statics for ω

The global comparative statics for ω are illustrated in Figure 3 and the intuition is as follows. When ω is low, myopia is sufficiently weak that the manager invests efficiently even with full disclosure. When ω rises above a threshold $\widetilde{\omega}$, $(\lambda^* = 1,\ \sigma^* = 1)$ is no longer sustainable and there is a trade-off. When β is very low (e.g., at β_a in Figure 3), the manager always chooses partial disclosure, and additional increases in ω reduce the partial disclosure level further. When β is very high (e.g., at β_b), the manager always chooses partial investment, and additional increases in ω reduce the partial investment level further. Recall that $\widetilde{\beta}$ is first decreasing and then increasing in ω. When β is moderately high (e.g., at β_c), and if also $\tilde{\beta}\,(X=1) > \beta > \tilde{\beta}\,(X=0)$, then when ω becomes sufficiently high (i.e. crosses the solid curve), $\widetilde{\beta}$ crosses back above β and so the manager switches to partial disclosure. When β is moderately low (e.g., at β_d), within the trade-off region, the manager chooses partial disclosure for low and high β, and partial investment for intermediate β. Considering all cases together, as with the other parameters, ω has a monotonic effect on investment, but a non-monotonic effect on disclosure.

Overall, Proposition 4 yields empirical predictions for how investment and disclosure vary cross-sectionally across firms. As is intuitive, and predicted by many other models, investment depends on corporate finance variables – it is increasing in the profitability of investment opportunities and decreasing in the manager's short-term concerns. More unique to our framework is that investment also depends on asset pricing variables. It decreases with the frequency of liquidity shocks and the information asymmetry suffered by small investors (which in turn depends on the noise of the public signal ρ and uncertainty Δ). Increases in these variables augment the cost of capital, and may induce the manager to switch from full investment to full disclosure.

The effects of corporate finance and asset pricing characteristics on disclosure policy are non-monotonic. One might expect that, since disclosure policy is a trade-off between investment and the cost of capital, better growth opportunities mean that investment dominates the trade-off, and so disclosure is lower. Instead, firms with intermediate growth opportunities disclose the least, because growth opportunities are sufficiently strong that the manager prefers full investment to full disclosure, but also sufficiently weak that he will only invest fully if disclosure is low. Firms with weak growth opportunities have high disclosure, because the cost of capital dominates the trade-off and so the manager implements the full-disclosure, partial-investment equilibrium. Firms with strong growth opportunities will have high disclosure for a different reason – the growth opportunity is sufficiently attractive that the firm will pursue it even with high disclosure.

For similar reasons, firms with moderate uncertainty Δ, moderate size β and frequency ϕ of liquidity shocks, and moderate signal noise ρ will have low disclosure, but firms with either high or low levels of these variables will feature high disclosure. For example, it may seem that, when uncertainty Δ rises, the manager will always disclose more in response. However, if it remains optimal to implement full investment, the manager must reduce disclosure to do so. Similarly, it may seem that, when ρ rises, the manager should disclose less as the signal is noisier. However, a rise in ρ makes the cost of capital more important, encouraging full disclosure. As Beyer, Cohen, Lys, and Walther's (2010) survey paper emphasizes, "it is necessary to consider multiple aspects of the corporate information environment in order to conclude whether it becomes more or less informative in response to an exogenous change." The effect of the manager's short-term concerns ω is more complex, but in all cases, disclosure is highest when short-term concerns are low, because the manager can disclose fully without suffering underinvestment.

The non-monotonic effects of firm characteristics on disclosure policy contrast with prior theories. Baiman and Verrecchia (1996) predict that a larger liquidity shock monotonically reduces disclosure. Gao and Liang (2013) predict that firms with higher growth opportunities disclose less, to encourage investors to acquire private information about the growth option and incorporate it into prices by trading, thus informing the manager. More generally, the model points to variables (e.g., g, β, ϕ, Δ, ρ) that empiricists should control for when studying the effect of a different variable (outside our model) on disclosure. In addition, it emphasizes that disclosure, investment, and the cost of capital are all simultaneously determined by underlying parameters, rather than affecting each other. As Beyer, Cohen, Lys, and Walther (2010) note: " 'equilibrium' concepts for the market for information defy a simplified view of cause and effect".

Lemma 3 derived monotonic partial derivatives for the excess cost of capital with respect to underlying parameters. However, the excess cost of capital depends also on σ, which depends non-monotonically on the same parameters. Thus, due to the endogenous response of disclosure policy, the overall effects of these parameters on the cost of capital is unclear. In contrast, Diamond and Verrecchia (1991) predict that the cost of capital is monotonically decreasing in information asymmetry and the magnitude of liquidity shocks.

3 Voluntary Disclosure

The analysis of Section 2 shows that, if the manager is able to commit to a disclosure policy, he may commit to partial disclosure even though this raises his cost of capital.

This section considers the case of voluntary disclosure, where the manager cannot commit to a disclosure policy. We now assume that he always possesses the signal

y, and chooses whether to disclose it. In reality, companies already have to produce copious amounts of information for tax or internal purposes, so the manager cannot commit to not having information. Thus, while the manager may announce a disclosure policy at $t = 0$, he may renege on it at $t = 2$. For example, he could implement a disclosure policy σ by using a private randomization device, e.g., spinning a wheel that has a fraction σ of "disclose" outcomes and $1 - \sigma$ of "non-disclose" outcomes, and disclosing the signal if and only if the wheel lands on "disclose". However, even if the device lands on "non-disclose", he may renege and disclose anyway.[12]

Since $P(G) > \widetilde{P}(\varnothing)$, the manager will choose to disclose the signal if it turns out to be good. Thus, the absence of disclosure reveals that $y = B$. The manager cannot claim that he is not disclosing to follow his pre-announced low-disclosure policy, because the market knows that he would have reneged on the policy if the signal were good. No news is bad news – the "unraveling" result of Grossman (1981) and Milgrom (1981).

The manager knows that he will always disclose at $t = 2$, either literally by disclosing $y = G$, or implicitly by not disclosing and the market inferring that $y = B$. Therefore, he will make his $t = 1$ investment decision assuming that $\sigma = 1$, i.e., choose $\lambda^* = r(1)$ irrespective of the preannounced policy. Thus, the voluntary disclosure model is equivalent to the mandatory disclosure model with $\sigma = 1$. Even if $\Pi(1, X) > \Pi(r(1), 1)$, and so the manager would like to commit to low disclosure, he is unable to do so. This result is stated in Proposition 5 below.

Proposition 5 *(Voluntary Disclosure): Consider the case in which the manager always possesses the signal y and has discretion over whether to disclose it at $t = 3$. The only Perfect Bayesian Equilibrium involves $\lambda^* = r(1)$ and $\sigma^* = 1$.*

Proposition 5 implies a potential role for government intervention. We now allow for the government to set a regulatory policy ζ at $t = 0$. At $t = 2$, with probability $1-\zeta$, the manager either cannot or chooses not to disclose due to the government's policy. For example, the government could ban disclosure (e.g., prohibit the disclosure of earnings more frequently than a certain periodicity).[13] Similarly, it could limit what type of information can be reported in official (e.g., SEC) filings, which investors may view as more truthful than information disseminated through, for example, company press releases. Alternatively, the government could audit disclosures with sufficient intensity

[12] In keeping with the literature on voluntary disclosure, the manager can never falsify the signal (e.g., release $y = G$ if the signal was $y = B$), and only has discretion on whether or not to disclose it.

[13] This is similar in spirit to the "quiet period" that precedes an initial public offering, which limits a firm's ability to disclose information.

that the manager chooses not to disclose: even if disclosure is always truthful, so there is no risk of a fine, responding to an audit is costly.

Now, when making his $t = 1$ investment decision, he knows that he will disclose at $t = 2$ only with probability ζ.[14] He will thus choose $\lambda^* = \lambda(\zeta)$. Therefore, if the government's goal is to maximize firm value to existing shareholders (i.e., the manager's payoff), it will choose a disclosure policy $\zeta = X$, thus implementing the $(\lambda^* = 1, \sigma = X)$ equilibrium. The government implements less disclosure than the manager would choose himself, since he is unable to commit to low disclosure. This conclusion contrasts some existing models (e.g., Foster (1979), Coffee (1984), Dye (1990), Admati and Pfleiderer (2000), Lambert, Leuz, and Verrecchia (2007)) which advocate that regulators should set a floor for disclosure, because firms have insufficient incentives to release information. It also contrasts recent increases in disclosure regulation, such as Sarbanes-Oxley, and is consistent with concerns that such regulation may reduce investment. If caps on disclosure are difficult to implement, a milder implication of our model is that government regulations to increase disclosure may have real costs.

However, government regulation may not maximize firm value. First, the policy that maximizes firm value varies from firm to firm. Even if all managers wish to implement full investment, the disclosure policy $\zeta = X \equiv \frac{g(1+\rho)}{\Omega\phi\rho(\Delta+g)}$ depends on firm characteristics. Regulation is typically economy-wide, rather than at the individual firm level. A policy of ζ will induce suboptimally low disclosure in a firm for which $X > \zeta$, since $\sigma = X$ is sufficient to implement full investment. In contrast, a policy of ζ will not constrain disclosure enough in a firm for which $X < \zeta$. The manager will invest only $r(\zeta) < 1$, although this is still higher than the benchmark of no regulation. Moreover, some managers will not wish to implement full investment if $\Pi(1, X) < \Pi(r(1), 1)$ for their firm. Thus, a regulation aimed at inducing full investment will be inefficient.

Second, the government's goal may not be to maximize firm value, but total surplus. In this case, it ignores the benefits of disclosure, since the investor's trading losses are a pure transfer to the speculator and do not affect total surplus. It only considers the cost of disclosure (reduced investment), and so will choose any $\zeta \in [0, X]$ to implement $\lambda^* = 1$. Such a policy will be suboptimal for the manager if $\Pi(1, X) < \Pi(r(1), 1)$.

Third, the government may have distributional considerations and aim to minimize informed trading profits and losses. One example is the SEC's focus on "leveling the playing field" between investors. Under this objective function, it will minimize the

[14] An alternative way to regulate may be to affect σ directly. For example, if the government allows greater discretion in accounting policies, managers have greater latitude for earnings management, and so earnings are a less informative signal.

investor's trading losses[15] and ignore investment, which is achieved with $\zeta = 1$. Thus will reduce firm value if $\Pi(1, X) > \Pi(r(1), 1)$.

These results are stated in Proposition 6 below.

Proposition 6 *(Regulation): If the government wishes to maximize firm value, it will set a policy of* $\zeta = X$ *if* $\Pi(1, X) > \Pi(r(1), 1)$ *and* $\zeta = 0$ *otherwise. If the government wishes to maximize total surplus, it will choose any* $\zeta \in [0, X]$*, which will implement* $\lambda^* = 1$*. If the government wishes to minimize the investor's trading losses, it will choose* $\zeta = 1$*, which will implement* $\lambda^* = r(1)$*.*

4 Conclusion

Existing theories study the costs and benefits of disclosing information. It may seem that, if soft information cannot be disclosed, firms should simply apply these theories to hard information. This paper reaches a different conclusion – the importance of soft information changes the optimal disclosure policy of hard information. While increasing the disclosure of hard information augments the total amount of information available to investors, and thus reduces the cost of capital, it also increases the amount of hard information disclosed relative to soft information. The manager may cut investment, to improve hard information at the expense of soft information. Even if the actual act of disclosing information is costless, a high-disclosure policy may be costly. Thus, real efficiency is non-monotonic in financial efficiency.

If the manager can commit to a disclosure policy, his optimal policy will vary according to the importance of growth opportunities versus the cost of capital. Investment depends not only on corporate finance variables but also asset pricing variables such as shareholders' liquidity needs and information disadvantage. The effect of firm characteristics on disclosure policies is subtle. As predicted by other models, the optimal disclosure policy depends on asset pricing variables such as the level of information asymmetry faced by investors. Here, the effect of such variables is non-monotonic. When information asymmetry is moderate, or investors suffer moderately frequent liquidity shocks, investment is more important than disclosure and so the manager reduces disclosure to pursue investment. When these parameters are high, disclosure dominates the trade-off and so the manager maximizes it; when these parameters are

[15] Note that minimizing the investor's trading losses is not the same as maximizing her objective function. The investor breaks even in all scenarios, since the initial stake that she requires takes into account her trading losses.

low, he is able to increase disclosure without suffering underinvestment. In addition, disclosure also depends on corporate finance variables such as the manager's horizon and the profitability of growth opportunities.

If the manager cannot commit to a disclosure policy, then even if a "high-investment, low-disclosure" policy is optimal, he may be unable to implement it as he will opportunistically disclose a good signal, regardless of the preannounced policy. Thus, there may be a role for government regulation to reduce disclosure.

The model suggests a number of avenues for future research. On the theory side, the paper has endogenized investment and disclosure, and studied how these decisions interplay with the manager's short-term concerns and the need to raise capital, which are taken as given. A potential extension would be to endogenize the manager's contract and the amount of capital raised, to study how these are affected by the same factors that drive investment and disclosure. Future studies could also relax the assumption that investors know the growth opportunities of a high-quality firm, in which case disclosure may have a role in signaling such opportunities.[16] In addition, we have assumed that the manager's disclosure is always truthful. If earnings management is possible, a good manager can avoid reporting a bad signal, but also a bad manager may report a good signal. While the former likely improves investment, the latter increases information asymmetry. Incorporating earnings management may deliver insights as to when discretion is beneficial and when it is harmful. On the empirical side, our study delivers new predictions on the real effects of disclosure on investment, on how investment depends on asset pricing variables, and on how the cost of capital and disclosure depend on corporate finance variables. In addition, while previous papers derived predictions on how the cost of capital and disclosure depend on asset pricing variables, our model predicts non-monotonic effects.

[16] In the current model, where only firm type is unknown, allowing for signaling (e.g. for managers to learn their type before setting disclosure policy) will simply lead to pooling equilibria as L-managers will mimic H-managers.

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A Appendix

Proof of Proposition 1

Fix any $\sigma \in [0,1]$. The quadratic $\Psi(\lambda,\sigma)$ has real roots if and only if the discriminant is non-negative, i.e.,

$$z(\sigma) \equiv \phi^2 \frac{\Delta^2}{g^2}\sigma^2 - 4\left(\frac{1}{\Omega} - \sigma\phi\right)\frac{1}{\Omega\rho} \geq 0. \tag{17}$$

The quadratic $z(\sigma)$ is a strictly convex function of σ with two roots. Since and $z(0) < 0$, it has one positive root which is given by:

$$Z \equiv \frac{g^2}{\Delta^2}\left[\frac{2}{\phi\Omega\rho}\sqrt{1+\rho\frac{\Delta^2}{g^2}} - \frac{2}{\phi\Omega\rho}\right].$$

Since $\sigma \in [0,1]$, for $z(\sigma) \geq 0$ (i.e., (17) to hold), σ must be weakly larger than the positive root Z. Thus, $\sigma \geq Z$ is necessary and sufficient for Ψ to have real roots.

Since $\Psi(0,\sigma) = \frac{1}{\Omega\rho} > 0$ and $\Psi'(0,\sigma) < 0$, Ψ may have up to two positive roots. One root, r, is such that $\Psi'(r,\sigma) < 0$. The second root, r', is such that $\Psi'(r',\sigma) \geq 0$. This second root, r', lies in $[0,1]$ if and only if $\Psi'(1,\sigma) \geq 0$, i.e.,:

$$\sigma \leq \frac{2g}{\Omega\phi(2g+\Delta)}. \tag{18}$$

However, further algebra shows that

$$X > Z > \frac{2g}{\Omega\phi(2g+\Delta)}. \tag{19}$$

Thus, if roots exist ($\sigma \geq Z$), (18) is violated and so the second root r' cannot lie in $[0,1]$. Therefore, the quadratic form of $\Psi(\lambda,\sigma)$ implies that there is at most one interior solution to the equation $\Psi(\lambda,\sigma) = 0$ for any $\sigma \in [0,1]$.

First, consider $\sigma \leq X$. Then $\Psi(1,\sigma) \geq 0$ by definition of X. Suppose there is $r' \in (0,1)$ such that $\Psi(r',\sigma) = 0$. The quadratic form of $\Psi(\lambda,\sigma)$ and $\Psi(0,\sigma) > 0$ implies that $\Psi'(1,\sigma) > 0$, which contradicts equation (19). Therefore, when $\sigma \leq X$, $\Psi(\lambda,\sigma) \geq 0$ (with equality only when $\lambda = 1$ and $\sigma = X$). Thus, the manager always wants to increase the investment level, and the unique equilibrium investment level is $\lambda^* = 1$.

Second, consider $\sigma > X$, in which case $\Psi(1,\sigma) < 0$. Then, when the market maker conjectures $\widehat{\lambda} = 1$, the manager has an incentive to deviate to a lower investment level. As a result, $\lambda = 1$ cannot be an equilibrium. Since $\Psi(0, \sigma) > 0$ and $\Psi(\lambda, \sigma)$ is continuous in λ, $\Psi(\lambda, \sigma) = 0$ has a solution $r \in [0, 1]$. As argued previously, we must have $\Psi'(r, \sigma) < 0$.

We now prove that $r(\sigma)$ is strictly decreasing and strictly concave. Recall that

$$\Psi(\lambda, \sigma) = \left(\frac{1}{\Omega} - \sigma\phi\right)\lambda^2 - \sigma\phi\frac{\Delta}{g}\lambda + \frac{1}{\Omega\rho},$$

and so we can calculate

$$\begin{aligned} \left.\frac{\partial\Psi}{\partial\lambda}\right|_r &= 2\left(\frac{1}{\Omega} - \sigma\phi\right)r - \sigma\phi\frac{\Delta}{g} < 0 \\ \left.\frac{\partial\Psi}{\partial\sigma}\right|_r &= -\phi\left(r^2 + \frac{\Delta}{g}r\right) < 0. \end{aligned}$$

Thus, the Implicit Function Theorem yields:

$$\frac{dr}{d\sigma} = -\frac{\partial\Psi/\partial\sigma}{\partial\Psi/\partial\lambda} < 0,$$

i.e., $r(\sigma)$ is strictly decreasing.

To prove strict convexity, note that

$$\frac{\partial^2 r}{\partial\sigma^2} = \frac{1}{(\partial\Psi/\partial\lambda)^2}\left\{-\left[\frac{\partial^2\Psi}{\partial\sigma\partial\lambda}\frac{\partial\lambda}{\partial\sigma} + \frac{\partial^2\Psi}{\partial\sigma^2}\right]\frac{\partial\Psi}{\partial\lambda} + \frac{\partial\Psi}{\partial\sigma}\left[\frac{\partial^2\Psi}{\partial\lambda^2}\frac{\partial\lambda}{\partial\sigma} + \frac{\partial^2\Psi}{\partial\lambda\partial\sigma}\right]\right\}.$$

Since $\partial^2\Psi/\partial\sigma^2 = 0$, plugging in $\frac{dr}{d\sigma} = -\frac{\partial\Psi/\partial\sigma}{\partial\Psi/\partial\lambda}$ yields:

$$\begin{aligned} &\frac{d^2 r}{d\sigma^2} > 0 \\ \Leftrightarrow\ &\frac{\partial^2\Psi}{\partial\lambda^2}\left(\frac{\partial\Psi/\partial\sigma}{\partial\Psi/\partial\lambda}\right) - 2\frac{\partial^2\Psi}{\partial\lambda\partial\sigma} > 0 \\ \Leftrightarrow\ &\left(\frac{1}{\Omega} - \sigma\phi\right)\frac{-\left(r^2 + \frac{\Delta}{g}r\right)}{2\left(\frac{1}{\Omega} - \sigma\phi\right)r - \sigma\phi\frac{\Delta}{g}} + \left(2r + \frac{\Delta}{g}\right) > 0. \end{aligned}$$

There are two cases to consider. First, if $\frac{1}{\Omega} - \sigma\phi \geq 0$, the above inequality automatically holds. Second, if $\frac{1}{\Omega} - \sigma\phi < 0$, we have

$$
\begin{aligned}
&\left(\frac{1}{\Omega} - \sigma\phi\right) \frac{-\left(r^2 + \frac{\Delta}{g} r\right)}{2\left(\frac{1}{\Omega} - \sigma\phi\right) r - \sigma\phi\frac{\Delta}{g}} + \left(2r + \frac{\Delta}{g}\right) > 0 \\
&\Leftrightarrow -\left(\frac{1}{\Omega} - \sigma\phi\right)\left(r^2 + \frac{\Delta}{g} r\right) + \left[2\left(\frac{1}{\Omega} - \sigma\phi\right) r - \sigma\phi\frac{\Delta}{g}\right]\left(2r + \frac{\Delta}{g}\right) < 0 \\
&\Leftrightarrow 3\left(\frac{1}{\Omega} - \sigma\phi\right) r^2 + \left(\frac{1}{\Omega} - \sigma\phi\right)\frac{\Delta}{g} r - 2\sigma\phi\frac{\Delta}{g} r - \sigma\phi\left(\frac{\Delta}{g}\right)^2 < 0.
\end{aligned}
$$

The last equation holds because all terms on the left-hand side are negative. Therefore, $r(\sigma)$ is strictly convex.

Now assume $X < 1$, and fix $\sigma > X$. We wish to show that $r(\sigma)$ is increasing in g, and decreasing in ω, ϕ, ρ, and Δ. Since $\sigma > X$ implies $\Psi'(r, \sigma) < 0$, the Implicit Function Theorem gives us that the signs of partial derivatives $\partial r / \partial g$, $\partial r / \partial \omega$, $\partial r / \partial \phi$, $\partial r / \partial \rho$, and $\partial r / \partial \Delta$ are the same as those of $\partial \Psi / \partial g$, $\partial \Psi / \partial \omega$, $\partial \Psi / \partial \phi$, $\partial \Psi / \partial \rho$, and $\partial \Psi / \partial \Delta$, respectively. By taking partial derivatives of Ψ (evaluated at $r(\sigma)$), we have

$$
\begin{aligned}
\frac{\partial \Psi}{\partial g} &= \sigma\phi\frac{\Delta}{g^2} r > 0, \\
\frac{\partial \Psi}{\partial \omega} &= -\frac{r^2 + \frac{1}{\rho}}{\omega^2} < 0, \\
\frac{\partial \Psi}{\partial \phi} &= -\sigma\left(r^2 + \frac{\Delta}{g} r\right) < 0, \\
\frac{\partial \Psi}{\partial \rho} &= -\frac{1-\omega}{\omega}\frac{1}{\rho^2} < 0.
\end{aligned}
$$

Therefore,

$$
\frac{\partial r}{\partial g} > 0, \frac{\partial r}{\partial \omega} < 0, \frac{\partial r}{\partial \phi} < 0, \text{ and } \frac{\partial r}{\partial \rho} < 0.
$$

Finally, analyzing equation (5) easily shows that X is increasing in g, and decreasing in ω, ϕ, ρ, and Δ.

Proof of Lemma 4

Since $\lambda^*(\sigma) = 1$ for all $\sigma \in [0, X]$, the manager's payoff becomes

$$
\Pi(\sigma) = \frac{1}{2}\left(R^H + g + R^L\right) - K - \beta\phi\frac{1}{2}(\Delta + g)\left[(1-\sigma)\frac{1}{2} + \sigma\frac{\rho}{1+\rho}\right],
$$

which is strictly increasing in σ as a higher σ reduces trading losses. Thus, the manager chooses the maximum σ in $[0, X]$, which is X.

Proof of Proposition 3

When choosing the disclosure policy, the manager compares the payoffs from $\sigma = 1$ (in which case $\lambda = r(1)$) and $\sigma = X$ (in which case $\lambda = 1$). Thus, the equilibrium is $(\lambda^* = r(1), \sigma^* = 1)$ if $\Pi(r(1), 1) > \Pi(1, X)$, and $(\lambda^* = 1, \sigma^* = X)$ otherwise.

The manager chooses $(\lambda^* = 1, \sigma^* = X)$ if $\Pi(1, X) - \Pi(r, 1) > 0$, i.e.,

$$(1-r)\left[\frac{1}{2} - \frac{1}{4}\beta\phi - \frac{1}{4}\beta\frac{1-\omega}{\omega}\right] + \frac{1-\omega}{\omega}\frac{\beta}{4\rho} + \frac{1}{4}\frac{1-\omega}{\omega}\beta r - \frac{1}{4}\beta\phi r - \frac{\beta\phi(\Delta)}{4g} > 0,$$

where we write r rather than $r(1)$ to economize on notation. Here, r can be solved from $\Psi(r, 1) = 0$, and $\Psi'(r, 1) < 0$. Since Ψ is not a function of β, the above inequality is equivalent to

$$1 - r > \beta\left\{\frac{1}{2}\phi\frac{\Delta+g}{g} - \frac{1-\omega}{\omega}\left[\frac{1}{2}\left(\frac{1}{\rho} - 1\right) + r\right]\right\}.$$

The term multiplied by β on the right-hand side is

$$\begin{aligned}
&\frac{1}{2}\phi\frac{\Delta+g}{g} - \frac{1-\omega}{\omega}\left[\frac{1}{2}\left(\frac{1}{\rho} - 1\right) + r\right] \\
>&\frac{1}{2}\phi\frac{\Delta+g}{g} - \phi\frac{\Delta+g}{g}\frac{\rho}{\rho+1}\left[\frac{1}{2}\left(\frac{1}{\rho} - 1\right) + r\right] \\
=&\phi\frac{\Delta+g}{g}\frac{\rho}{\rho+1}\left[1 - r\right] \\
>&0.
\end{aligned}$$

The first inequality is due to the condition $X < 1$. As a result,

$$\widetilde{\beta} = \frac{1-r}{\frac{1}{2}\phi\frac{\Delta+g}{g} - \frac{1-\omega}{\omega}\left[\frac{1}{2}\left(\frac{1}{\rho} - 1\right) + r\right]} > 0.$$

Since the denominator of $\widetilde{\beta}$ is strictly greater than $\frac{1-\omega}{\omega}\frac{1}{X}(1-r)$, we have $\widetilde{\beta} < \frac{\omega}{1-\omega}X$. Thus, the manager strictly prefers $(\lambda^* = 1, \sigma^* = X)$ if and only if $\beta < \widetilde{\beta}$.

When $X < 1$, to derive the comparative statics of $\tilde{\beta}$, we first define

$$\chi(\beta) = (1-r) - \beta\left\{\frac{1}{2}\phi\frac{\Delta+g}{g} - \frac{1-\omega}{\omega}\left[\left(\frac{1}{2}\frac{1}{\rho} - 1\right) + r\right]\right\}.$$

It is clear that $\chi\left(\widetilde{\beta}\right)=0$ and $\chi'\left(\widetilde{\beta}\right)<0$. Thus, the signs of $\partial\widetilde{\beta}/\partial g$, $\partial\widetilde{\beta}/\partial\phi$, $\partial\widetilde{\beta}/\partial\rho$, and $\partial\widetilde{\beta}/\partial\omega$ are the same as those of $\partial\chi/\partial g$, $\partial\chi/\partial\phi$, $\partial\chi/\partial\rho$, and $\partial\chi/\partial\omega$ (evaluated at $\widetilde{\beta}$).

First, we show that $\partial\chi/\partial g>0$, so $\partial\widetilde{\beta}/\partial g>0$.

$$\begin{aligned}
&\partial\chi/\partial g=\left(\widetilde{\beta}\frac{1-\omega}{\omega}-1\right)\frac{\partial r}{\partial g}+\frac{1}{2}\widetilde{\beta}\phi\frac{\Delta}{g^2}>0\\
\Leftrightarrow&\frac{\frac{1-\omega}{\omega}\left[\frac{1}{2\rho}+\frac{1}{2}\right]r-\frac{1}{2}\phi\frac{\Delta+g}{g}r}{\phi\frac{\Delta}{g}-2\left[\frac{1-\omega}{\omega}-\phi\right]r}+\frac{1}{2}(1-r)>0\\
\Leftrightarrow&(r-1)^2>0.
\end{aligned}$$

The last inequality is automatic, because $r<1$ when $X<1$.

Second, we also show $\partial\chi/\partial\phi<0$, so $\partial\widetilde{\beta}/\partial\phi<0$.

$$\begin{aligned}
&\partial\chi/\partial\phi<0\\
\Leftrightarrow&\left(\widetilde{\beta}\frac{1-\omega}{\omega}-1\right)\frac{\partial r}{\partial\phi}-\frac{1}{2}\widetilde{\beta}\frac{\Delta+g}{g}<0\\
\Leftrightarrow&\left[-\left(\frac{1-\omega}{\omega}-\phi\right)r+\frac{1-\omega}{\omega}\frac{1}{\rho}\right]\left(\frac{\Delta}{g}+r\right)\\
&\quad-\left[-\left(\frac{1-\omega}{\omega}-\phi\right)r+\frac{1-\omega}{\omega}\frac{1}{\rho}\frac{1}{r}\right]\left(\frac{\Delta}{g}+1\right)<0.
\end{aligned}$$

The final inequality is true because all of the following inequalities hold:

$$\begin{gathered}
-\left(\frac{1-\omega}{\omega}-\phi\right)r+\frac{1-\omega}{\omega}\frac{1}{\rho}\frac{1}{r}>-\left(\frac{1-\omega}{\omega}-\phi\right)r+\frac{1-\omega}{\omega}\frac{1}{\rho},\\
-\left(\frac{1-\omega}{\omega}-\phi\right)r+\frac{1-\omega}{\omega}\frac{1}{\rho}\frac{1}{r}>0\quad\text{(because }\Psi'(r,1)<0\text{), and}\\
\frac{\Delta}{g}+1>\frac{\Delta}{g}+r.
\end{gathered}$$

Then, we show $\partial\chi/\partial\rho<0$, so $\partial\widetilde{\beta}/\partial\rho<0$.

$$\partial\chi/\partial\phi=\left(\widetilde{\beta}\frac{1-\omega}{\omega}-1\right)\frac{\partial r}{\partial\rho}-\widetilde{\beta}\frac{1-\omega}{2\omega}\frac{1}{\rho^2}.$$

Hence,

$$\begin{aligned} &\partial\chi/\partial\phi < 0 \\ \Leftrightarrow & -\left(\frac{1-\omega}{\omega}-\phi\right)(1-r)^2 < 0. \end{aligned}$$

Finally, we show that $\partial\chi/\partial\omega$ depends on ω, so the sign of $\partial\widetilde{\beta}/\partial\omega$ depends on ω.

$$\partial\chi/\partial\omega = \left(\widetilde{\beta}\frac{1-\omega}{\omega}-1\right)\frac{\partial r}{\partial\omega} - \widetilde{\beta}\frac{1}{\omega^2}\left[\frac{1}{2}\left(\frac{1}{\rho}-1\right)+r\right].$$

When ω is small, so that X is close to 1, we have $\widetilde{\beta}\frac{1-\omega}{\omega}-1 \to 0$ and $r \to 1$. Thus, $\partial\chi/\partial\omega < 0$. When $\omega \to 1$, $r \to 0$ (from equation (4)). Then,

$$\begin{aligned} &\partial\chi/\partial\omega > 0 \\ \Leftrightarrow & \frac{-\frac{1-\omega}{\omega}\left(\frac{1}{2\rho}+\frac{1}{2}\right)+\frac{1}{2}\phi\frac{\Delta+g}{g}}{\phi\frac{\Delta}{g}-2\frac{1-\omega}{\omega}r}\left[r^2+\frac{1}{\rho}\right] \\ & -(1-r)\left[\frac{1}{2}\left(\frac{1}{\rho}-1\right)+r\right] > 0. \end{aligned}$$

The left-hand side converges to $\frac{1}{2\rho}\frac{g}{\Delta}+\frac{1}{2} > 0$.

Proof of Proposition 4

We first provide more precise details on the global comparative statics of Proposition 4.

(i) Comparative statics for g:

- ***(i-a)*** If $\beta > \lim_{g\to\infty}\tilde{\beta}$, $\sigma^* = 1$ and $\lambda^* = r(1)$, which increases as g increases.
- ***(i-b)*** If $0 < \beta < \Omega$ and $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} > 1$, $\sigma^* = 1$ and $\lambda^* = r(1)$ for low levels of g. Once g rises above a threshold, σ^* falls discontinuously to X, and λ^* jumps discontinuously to 1. As g increases further, σ^* keeps increasing to 1 (for g such that $X \geq 1$), while $\lambda^* = 1$.
- ***(i-c)*** If $0 < \beta < \lim_{g\to\infty}\tilde{\beta}$ and $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} \leq 1$, $\sigma^* = 1$ and $\lambda^* = r(1)$ for low levels of g. Once g rises above a threshold, σ^* falls discontinuously to X, and λ^* jumps discontinuously to 1. As g increases further, σ^* keeps increasing but remains below 1, while $\lambda^* = 1$.

(ii) Comparative statics for Δ:

(ii-a) If $\beta > \lim_{\Delta \to 0} \tilde{\beta}$, $\sigma^* = 1$ and $\lambda^* = r(1)$, which increases as Δ decreases.

(ii-b) If $0 < \beta < \Omega$ and $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} \leq 1$, $\sigma^* = 1$ and $\lambda^* = r(1)$ for high levels of Δ. Once Δ drops below a threshold, σ^* falls discontinuously to X, and λ^* jumps discontinuously to 1. As Δ decreases further, σ^* keeps increasing but remains below 1, while $\lambda^* = 1$.

(ii-c) If $0 < \beta < \lim_{\Delta \to 0} \tilde{\beta}$ and $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} > 1$, $\sigma^* = 1$ and $\lambda^* = r(1)$ for high levels of Δ. Once Δ drops below a threshold, σ^* falls discontinuously to X, and λ^* jumps discontinuously to 1. As Δ decreases further, σ^* keeps increasing to 1 (for Δ such that $X \geq 1$), while $\lambda^* = 1$.

(iii) Comparative statics for ϕ:

(iii-a) If $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} < 1$ and $\beta > \Omega$, then for small ϕ, the equilibrium is always $(\lambda^* = 1,\ \sigma^* = 1)$. Once ϕ rises above a threshold, the equilibrium is $(\lambda^* = r(1),\ \sigma^* = 1)$. Investment falls continuously; further increases in ϕ reduce λ^*, but σ^* is unaffected.

(iii-b) If $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} < 1$ and $\beta < \tilde{\beta}(\phi = 1)$, then for small ϕ, the equilibrium is $(\lambda^* = 1,\ \sigma^* = 1)$. Once ϕ rises above a threshold, the equilibrium is $(\lambda^* = 1,\ \sigma^* = X)$. Disclosure falls continuously; further increases in ϕ reduce σ^*, but λ^* is unaffected.

(iii-c) If $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} < 1$ and $\beta \in \left(\tilde{\beta}(\phi = 1), \Omega\right)$, then for small ϕ, the equilibrium is $(\lambda^* = 1,\ \sigma^* = 1)$. Once ϕ rises above a threshold, the equilibrium is $(\lambda^* = 1,\ \sigma^* = X)$. Disclosure falls continuously; further increases in ϕ reduce σ^*, but λ^* is unaffected. Once ϕ rises above a second threshold, the equilibrium switches to $(\lambda^* = r(1),\ \sigma^* = 1)$. Disclosure rises discontinuously and investment falls discontinuously; further increases in ϕ reduce λ^* but have no effect on σ^*.

(iii-d) If $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} \geq 1$, $X \geq 1$ for all ϕ. Then the equilibrium is always $(\lambda^* = 1,\ \sigma^* = 1)$.

(iv) Comparative statics for ρ:

(iv-a) If $\frac{1}{\Omega}\frac{2}{\phi}\frac{g}{\Delta+g} < 1$ and $\beta > \Omega$, then for small ρ, the equilibrium is always $(\lambda^* = 1,\ \sigma^* = 1)$. Once ρ rises above a threshold, the equilibrium is $(\lambda^* = r(1),\ \sigma^* = 1)$. Investment falls continuously; further increases in ρ reduce λ^*, but σ^* is unaffected.

(iv-b) If $\frac{1}{\Omega}\frac{2}{\phi}\frac{g}{\Delta+g} < 1$ and $\beta < \tilde{\beta}(\rho = 1)$, then for small ρ, the equilibrium is $(\lambda^* = 1,\ \sigma^* = 1)$. Once ρ rises above a threshold, the equilibrium is $(\lambda^* = 1,\ \sigma^* = X)$. Disclosure falls continuously; further increases in ρ reduce σ^*, but λ^* is unaffected.

(iv-c) If $\frac{1}{\Omega}\frac{2}{\phi}\frac{g}{\Delta+g} < 1$ and $\beta \in \left(\tilde{\beta}(\rho = 1), \Omega\right)$, then for small ρ, the equilibrium is $(\lambda^* = 1,\ \sigma^* = 1)$. Once ρ rises above a threshold, the equilibrium is $(\lambda^* = 1,\ \sigma^* = X)$. Disclosure falls continuously; further increases in ρ reduce σ^*, but λ^* is unaffected. Once ρ rises above a second threshold, the equilibrium switches to $(\lambda^* = r(1),\ \sigma^* = 1)$. Disclosure rises discontinuously and investment falls discontinuously; further increases in ρ reduce λ^* but have no effect on σ^*.

(iv-d) If $\frac{1}{\Omega}\frac{2}{\phi}\frac{g}{\Delta+g} \geq 1$, $X \geq 1$ for all ρ. Then the equilibrium is always $(\lambda^* = 1,\ \sigma^* = 1)$.

(v) Comparative statics for ω. Let $\underline{\beta}$ denote the minimum $\tilde{\beta}$ over all ω such that $X \leq 1$:

(v-a) If $\beta < \underline{\beta}$, then for low ω, the equilibrium is $(\lambda^* = 1, \sigma^* = 1)$; once ω rises above a threshold, the equilibrium is $(\lambda^* = 1,\ \sigma^* = X)$. Disclosure falls continuously; further increases in ω lower σ^* but have no effect on λ^*.

(v-b) If $\beta > \max\left\{\tilde{\beta}(X = 1), \tilde{\beta}(X = 0)\right\}$, then for low ω, the equilibrium is $(\lambda^* = 1, \sigma^* = 1)$. Once ω rises above a threshold, the equilibrium is $(\lambda^* = r(1), \sigma^* = 1)$. Investment falls continuously; further increases in ω lower λ^*, but σ^* is unaffected.

(v-c) If $\tilde{\beta}(X = 1) > \beta > \tilde{\beta}(X = 0)$, then, in addition to the effects in part (b), once ω rises above a second threshold, the equilibrium switches to $(\lambda^* = 1,\ \sigma^* = X)$. Investment rises discontinuously and disclosure falls discontinuously; further increases in ϕ lower σ^* but have no effect on λ^*.

(v-d) If $\beta \in \left(\underline{\beta}, \min\{\tilde{\beta}(X = 1), \tilde{\beta}(X = 0)\}\right)$, then for low ω, the equilibrium is $(\lambda^* = 1, \sigma^* = 1)$. Once ω rises above a threshold, the equilibrium is $(\lambda^* = 1,\ \sigma^* = X)$. Disclosure falls continuously; further increases in ω lower σ^*, but λ^* is unaffected. Once ω rises above a second threshold, the equilibrium switches to $(\lambda^* = r(1), \sigma^* = 1)$. Disclosure rises discontinuously and investment falls discontinuously; further increases in ω lower λ^* but have no effect on σ^*. Once ω rises above a third threshold, the equilibrium switches to $(\lambda^* = 1,\ \sigma^* = X)$. Investment rises discontinuously and disclosure falls discontinuously; further increases in ϕ lower σ^* but have no effect on λ^*.

We now prove the proposition. We start with part (i), the global comparative statics with respect to g; the effect of Δ in part (ii) is exactly the opposite since Δ and g appear together as the ratio $\frac{\Delta+g}{g}$ in both X and $\tilde{\beta}$. From Proposition 3, $\tilde{\beta}$ is strictly increasing in g for $X < 1$. For part (i-a), if $\beta > \lim_{g\to\infty} \tilde{\beta}$, $\beta > \tilde{\beta}$ for all g. Then by Proposition 3, $\sigma^* = 1$ for all g, and $\lambda^* = r(1)$, which is strictly increasing in g.

For part (i-b), since $\tilde{\beta} = 0$ when $g = 0$, when g is small, $\beta > \tilde{\beta}$, and so the equilibrium is $(\lambda^* = r\,(1)\,,\, \sigma^* = 1)$. As g increases, the equilibrium remains $(\lambda^* = r\,(1)\,,\, \sigma^* = 1)$ but the investment level $r\,(1)$ is increasing. When g hits the point at which $\tilde{\beta} = \beta$, the equilibrium jumps to $(\lambda^* = 1,\ \sigma^* = X)$, so investment rises and disclosure falls. As g continues to increase, λ^* is constant at 1, while σ^* increases but remains strictly below 1: since $X < 1$, we can never have full disclosure alongside full investment.

Part (i-c) is similar to part (i-b), except that $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} > 1$. In this case, there exists a threshold g' such that, when $g \geq g'$, (9) is satisfied and we have $X \geq 1$. Note that $X = 1 \Leftrightarrow \tilde{\beta} = \Omega$. If $\beta \geq \Omega$, then we always have $\beta > \tilde{\beta}$ and full disclosure. When $g < g'$, the equilibrium is $(\lambda^* = r\,(1)\,,\, \sigma^* = 1)$. As g rises, $\lambda^* = r\,(1)$ rises. When g crosses above g', we now have full investment as well as full disclosure: the equilibrium becomes $(\lambda^* = 1,\ \sigma^* = 1)$. If $\beta \in (0, \Omega)$, then for low g, we have the partial investment equilibrium $(\lambda^* = r\,(1)\,,\, \sigma^* = 1)$. As g rises, σ^* remains constant at 1 and the partial investment level $r\,(1)$ rises, until $\tilde{\beta}$ crosses above β and we move to the full partial disclosure equilibrium $(\lambda^* = 1,\ \sigma^* = X)$. Note this crossing point for g is below g', because $\beta < \Omega$. As g continues to increase, λ^* is constant at 1 and σ^* rises. When g crosses above g', we have $X \geq 1$ so σ^* rises to 1. Unlike in the $\frac{1}{\Omega}\frac{1}{\phi}\frac{1+\rho}{\rho} \leq 1$ case, we can have full disclosure alongside full investment.

We now turn to part (iii). In part (iii-a), $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} \geq 1$, (9) is satisfied for all ϕ. Thus, we always have $X \geq 1$, which yields the equilibrium $(\lambda^* = 1,\ \sigma^* = 1)$. When $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} < 1$, there are several cases to consider. In part (iii-b), $\beta \geq \Omega$, then $\beta \geq \tilde{\beta}$ always and so we have partial investment. In part (iii-c), $\beta \leq \tilde{\beta}\,(\phi = 1)$, then $\beta \leq \tilde{\beta}$ always and so we always have partial disclosure. Finally, in part (iii-d) $\beta \in \left(\Omega, \tilde{\beta}\,(\phi = 1)\right)$, for small ϕ, we have $X \geq 1$, so the equilibrium is $(\lambda^* = 1, \sigma^* = 1)$. When ϕ rises so that X crosses below 1, then $\tilde{\beta}$ crosses above Ω and so we have $\beta < \tilde{\beta}$, which yields partial disclosure. After ϕ reaches a threshold, then $\tilde{\beta}$ falls below β and so we move to partial investment.

The proof of part (iv) is very similar, except that the cases of $\frac{1}{\Omega}\frac{1+\rho}{\rho}\frac{g}{\Delta+g} \lesseqgtr 1$ are replaced by $\frac{1}{\Omega}\frac{2}{\phi}\frac{g}{\Delta+g} \lesseqgtr 1$, and $\tilde{\beta}(\phi = 1)$ is replaced by $\tilde{\beta}(\rho = 1)$.

Finally, we prove part (v). When ω is sufficiently small that $X \geq 1$, the equilibrium is $(\lambda^* = 1, \sigma^* = 1)$. When ω is sufficiently large, $X < 1$. The remainder of this proof will focus on which equilibrium is chosen when $X < 1$. Proposition 3 shows that when ω is small so that X is close to 1 (while remaining below 1), $\tilde{\beta}$ is decreasing in ω. When ω is large, $\tilde{\beta}$ is increasing in ω. If $\underline{\beta}$ denotes the minimum $\tilde{\beta}$ over all ω such that $X \leq 1$, then $\underline{\beta} < \min\left\{\tilde{\beta}(X = 1), \tilde{\beta}(X = 0)\right\}$.

For part (v-a), when $\beta < \underline{\beta}$, then $\beta < \tilde{\beta}$. Thus, when $X < 1$, we always have the partial disclosure equilibrium of $(\lambda^* = 1, \sigma^* = X)$. For part (v-b), when $\beta > \max\left\{\tilde{\beta}(X = 1), \tilde{\beta}(X = 0)\right\}$, $\beta > \tilde{\beta}$. Thus, when $X < 1$, we always have the partial investment equilibrium of $(\lambda^* = r(1), \sigma^* = 1)$. For part (v-c), when $\beta > \min\left\{\tilde{\beta}(X = 1), \tilde{\beta}(X = 0)\right\}$, then when ω rises sufficiently for X to cross below 1, $\beta > \tilde{\beta}$ and so we have the partial investment equilibrium of $(\lambda^* = r(1), \sigma^* = 1)$. If we also have $\tilde{\beta}(X = 1) > \beta > \tilde{\beta}(X = 0)$, then once ω crosses a second threshold, then $\tilde{\beta}$ crosses below β and so we move to the partial disclosure equilibrium of $(\lambda^* = 1,\ \sigma^* = X)$. For part (v-d), when $\beta \in \left(\underline{\beta}, \min\{\tilde{\beta}(X = 1), \tilde{\beta}(X = 0)\}\right)$, then when ω rises sufficiently for X to cross below 1, then $\beta < \tilde{\beta}$ and so we have the partial disclosure equilibrium of $(\lambda^* = 1,\ \sigma^* = X)$. Since $\tilde{\beta}$ is decreasing in ω for low ω, When ω crosses a second threshold, then $\tilde{\beta}$ crosses below β and so we move to partial disclosure. Since $\tilde{\beta}$ is increasing in ω for high ω, when ω crosses a third threshold, then $\tilde{\beta}$ crosses back above β and so we move to partial investment.