Free Shipping and Repeat Buying on the Internet: Theory and Evidence

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Abstract

Free shipping is widely regarded as the most effective marketing tool in e-tailing. We model a rational, cost-minimizing shopper who responds to prices and shipping policies set by the e-tailer. A base scenario compares the optimal shopping policy under free and fixed fee shipping. A more complex case of value-contingent free shipping (e.g. shipping is free for orders over $X) explores how prices and the free shipping threshold interact to affect the optimal policy.

Compared to free shipping, shipping fees increase the rational shopper's optimal purchase quantities per visit, and the optimal elapsed time between visits. Contingent free shipping induces an iso-cost curve where different combinations of price and threshold keep long run average shopping costs constant for the rational shopper. Furthermore, the price-threshold relationship is shown be inverted-U shaped. This implies (1) the initial price level dictates whether prices should be raised or lowered when a site lowers the threshold for free shipping, and (2) price dispersion for homogenous goods increases when the threshold is lowered.

Key Words: *Free Shipping, Internet, Retailing, Shopping Behavior*

1 Introduction

Internet-based shopping is the fastest growing sector in US retailing with sales exceeding $110 billion for 2004 (Forrester Research). E-tailers have a number of marketing tools at their disposal and in many instances specific tactics (such as price cuts or coupons) and their consequences are largely analogous to those in offline settings. Free shipping, however, represents an aspect of e-tailing for which there is often no clear correspondence with offline practice. One may argue that "shipping costs" substitute for costs of time and travel, yet it is unclear that consumers view them in this way.[1] Shipping discounts appear vital for attracting customers and generating sales as approximately sixty percent of online retailers cite "free shipping with conditions" as their most successful marketing tool for driving business.[2] This well-documented "success" of free shipping and the relative paucity of research on the topic motivates our study. We model a rational cost-minimizing shopper's response to price and shipping policies to develop insights into how they affect shopping behavior. Model implications are tested using comScore data on repeat purchase behavior.

Shipping Fees and Shopping Behavior. The behavioral mechanism is unclear, yet managers are confident that shipping schedules have a substantial impact on consumer behavior. Bizrate.com CEO Chuck Davis noted that " ... (free shipping) with conditions turned out to be the defining characteristic of the 2002 holiday season" adding "This is not surprising considering that the majority of online buyers said that shipping costs prevent them from buying more online." A January 2004 survey by NetIQ Corp concurs: " ... shipping and handling costs trigger 52% of the abandonment of online shopping carts." (DMA 2004). Clearly, anecdotal evidence suggests that: (a) consumers are highly responsive to shipping policies, (b) there is room for improvement in how policies are crafted, and (c) little is known about consumer trade-offs that lead to the observed behaviors.

As the e-tailing sector grows, shipping fee structures will receive more attention (Cox 2002; Miller and Franco 2002). Absent formal research insight into how free shipping works, many e-tailers including Amazon.com experiment with different shipping policies.[3] A tangi-

[1]Travel costs are incurred prior to product selection, whereas shipping fees are incurred at the conclusion of the site visit.

[2]There are a variety of sources for this figure including shop.org/BizRate.com (2002 and 2003 Online Holiday Mood Study). The data on which this estimate is based is provided directly from online retailers.

[3]Amazon.com began experimenting with free shipping in 2002 and made it available for customers who

ble illustration and point of motivation for our study is provided by Lewis, Singh and Fay (2005). They estimate empirical choice and demand models on the order value and purchase incidence decisions of shoppers at an online retailer selling non-perishable grocery and drugstore items. Table 1 reproduces data from Table 2 in Lewis et al (2005).

[**Table 1 About Here**]

The rows of Table 1 distinguish two different kinds of fee structure — free shipping and contingent free shipping. The first three columns indicate the shipping fee attached to orders falling in different size classes. The final column shows the average order value under the corresponding shipping fee regime. The data are consistent with the idea that consumers seek to ammortize higher shipping fees by purchasing larger quantities per site visit. Of course, data in Table 1 do not control directly for selection effects (different regimes attract different kinds of shoppers) and we return to this point later.

Contribution and Caveats. We develop a model of a rational cost minimizing shopper, ordering non-durable products from an Internet retailer. When the products run out, the shopper returns to the site. Conditional on needs that trigger the site visit, the consumer makes purchase quantity decisions in response to the price and shipping policies set by the retailer. Analytical expressions for the optimal shopping policy under different shipping fee structures are obtained. We find the following:

- Consistent with the empirical work in Lewis et al (2005), imposition of a shipping fee induces higher purchase quantities per visit from the rational shopper. The rational shopper also increases the elapsed time between site visits.

- Contingent free shipping (use of a "free shipping threshold") creates tension between price and the shipping fee. Specifically, we derive an "iso-cost curve" of combinations of price and threshold that impose identical long run average shopping costs on rational consumers. It behaves as follows. As prices are increased, free shipping thresholds should first increase then decrease in order to leave a rational consumer equally well

place orders above a certain threshold. Initially, this was $99 (of eligible products) and was subsequently lowered to $49 and then to $25 (see Jung 2003). In the Discussion section we report Amazon's latest shipping policy experiment — Amazon Prime — "all-you-can-eat" express shipping.

off.[4] Typically, price increases are bad news for consumers, everything else constant. With repeat buying on the Internet however, a price increase has negative and positive effects. The negative effect of higher prices is that on average, purchasing costs will increase. The positive effect is that the average probability of exceeding the free shipping threshold goes up, leading to a reduction in the average shipping fee incurred.

- Prices and free shipping thresholds interact in a subtle way. Whether or not a reduction in a free shipping threshold should be accompanied by a price increase or decrease is shown to depend on the existing price level. We extend this result to show that price dispersion for homogenous goods should be greater when free shipping thresholds are lowered. An exogenous change in the free shipping threshold at a leading e-tailer is used to examine this implication.

The paper represents the first attempt (to our knowledge) to derive closed form characterizations of the optimal shopping policy for consumers facing an e-tailer who sets price and shipping policies. The contribution therefore lies in an analytical model to explore consumer behavior in this setting. Under value-contingent shipping, the retail price and free-shipping threshold interact to impact consumers' ordering policy in a complex yet predictable way. The model is intended to cover common purchasing contexts where consumers visit, store inventory, and then repeat visit once the inventory is depleted. Thus, it is not suited to address all classes of Internet shopping behavior. The model is firmly grounded in the behavior of the *consumer*; we do not explicitly address what is optimal for the firm. The model leads to testable implications, some of which are examined with exploratory empirical analysis and reference to other published work.

The remainder of the paper is organized as follows. The next section introduces the model. We compare and contrast a shopping context in which shipping costs are largely irrelevant (shipping is either free or always charged) with one where they are contingent: If a certain value threshold is reached, consumers receive free shipping. Exploratory empirical analysis follows in a separate section and the paper concludes with a discussion of the findings and avenues for future research.

[4] A graphical representation shows an inverted-U where price is on the x-axis and the free shipping threshold that holds total cost constant is on the y-axis.

2 Background and Model

We offer a brief review of related work and then develop the model. Assumptions are motivated with respect to existing literature and institutional details of Internet shopping. First, we analyze a base case of free shipping and fixed fees. We then analyze the contingent free shipping case, where the shopper pays a fixed fee when the total value of the order falls below a certain threshold, and obtains free shipping otherwise.

2.1 Related Work

As noted in the Introduction, there are many anecdotes regarding the effects of free shipping and the difficulty in implementing the “right” schedule, yet formal research is scarce. Morwitz, Greenleaf and Johnson (1998) study partitioned pricing schemes of the following kind: (1) “$82.90 including shipping and handling” and (2) “$69.95 plus $12.95 shipping and handling”. They find that when prices are partitioned as in (2) consumers can have higher demand. They are less likely to recall the full total cost but instead more likely to focus on the product cost alone. This might imply that E-tailers who charge for shipping should position it as an “add on” cost. Schindler, Morrin and Bechwati (2005) explore this finding further and show preference for bundled shipping fees is related to an individual different variable they label “shipping charge skepticism.” Skeptics with access to external reference prices prefer bundled prices. These articles do not address free shipping thresholds.

Hess, Chu and Gerstner (1996) analyze the strategic use of non-refundable shipping fees to mitigate returns by catalog shoppers. Consistent with the theory, empirical analysis suggests that fees increase with the value of the merchandise ordered, even when costs remain constant. Cao and Zhao (2004) study customer perceptions of inventory fulfillment but do not address free shipping directly. They find that satisfaction with delivery is moderated by overall satisfaction with prices.

Lewis (2005) estimates how shipping fees affect order size and site traffic. He finds that free shipping brings more customers, but lower average order sizes (compared to other schedules where shipping is not free). Incentives that offer lower shipping fees on larger orders also increase the order value. Lewis, Singh and Fay (2005) find free shipping policies increase order sizes by approximately 10% on average and up to 35% for the most responsive customer segment.

2.2 Context and Assumptions

We model a risk neutral shopper buying products on the Internet. The shopper first visits a site and then makes a purchase decision for goods that are bought repeatedly over time. Thus, the model could apply to the purchase of groceries or any other consumable goods for which there is repeat. These are also goods for which the shopper incurs positive storage costs for any inventory on hand. Repeat visits to the site are trigged by low inventory levels (which may be set to zero without loss of generality). The structure is familar as it builds on the classic model of Golabi (1985) and recent extensions (e.g., Ho, Tang and Bell 1998).

Prior to visiting the website, the shopper plans to purchase some quantity of the product in question. A fixed quantity Q is selected on the basis of the prevailing price p.[5] While the price is known deterministically, the actual quantity of goods purchased is a random variable. The randomness expresses the notion that upon visiting the site, particular marketing initiatives at the point of purchase (e.g., recommendations, pop-ups, etc.) could influence the final quantity bought. Considering the nth visit by the shopper, the actual quantity purchased is therefore $Q_n = Q + \epsilon_n$, where ϵ_n is a random element that captures the effect of these marketing initiatives. The random shocks $\epsilon_n, n \geq 1$ are iid with mean zero and variance σ^2. Φ denotes the cumulative distribution function associated with ϵ_n.

The shopper has a known consumption rate r for goods under consideration. These are consumed repeatedly. That is, when groceries (or other replenishable products) run out, the consumer returns to the site.[6] Time between successive visits is determined as follows. If the consumer purchases Q_n at the nth visit, then the elapsed time until the next purchase occasion is given by Q_n/r. Once at the website, the shopper incurs a visit-specific transaction cost k (searching for product information, typing in payment details and so forth). Moreover, the shopper incurs inventory holding costs h per unit time on all products actually bought. This implies that the inventory cost incurred subsequent to the n^{th} visit is given by $h \cdot Q_n/2 \cdot Q_n/r$. That is, the holding cost h multiplied by the expected inventory

[5] Ho, Tang and Bell (1998) employ a similar model structure but allow the (stochastic) price to follow a probability distribution with n mass points. In our model, the price is known by the shopper without cost — since on the Internet there is no fixed cost of travel to visit a store and learn precise price information.

[6] Implicitly, we assume that shopping at the site is triggered by zero "inventory" of the product, however it is straightforward to alter the model to consider the possibility that future site visits are triggered by a non-zero threshold. Doing so has no effect on qualitative insights.

on hand, $Q_n/2$, multiplied by the elapsed time to the next visit, Q_n/r. The final element of cost is product expenditures which are equal to pQ_n.

Given these assumptions and a context of repeat buying for replenishable items, we can characterize the optimal shopping policy. We determine the optimal purchase quantity per site visit, the optimal elapsed time between visits and the total long run average cost of shopping. We subsequently augment the basic model set up with a cost of shipping per trip, S_n. Regardless of the particular shipping policy adopted by the site (free shipping, fixed fees, contingent fees, etc.) the objective of the shopper is always to choose the purchase quantity Q^* such that the total long-run average relevant cost per unit time is minimized.

2.3 Model Analysis

We analyze two related classes of problem. In the first, shipping costs S_n have rather obvious effects on the optimal ordering policy. This serves as the base case and covers free shipping and fixed charges (independent of purchase value). The second class is value-contingent shipping fees. Many sites (including the one represented in Table 1) offer free shipping when purchases exceed a certain threshold.[7]

Free Shipping and Fixed Fees

Given we analyze repeat buying, we start with the long run average cost per unit time to be minimized by the shopper. This has two components: (1) the expected relevant cost per unit time (i.e., the cost incurred on a site visit), and (2) the expected elapsed time between successive visits. The expression for the optimal purchase quantity is then obtained by straightforward differentiation. Define C_n to be the cost incurred on the n^{th} visit

$$C_n = k + S_n + pQ_n + \frac{h}{2r}Q_n^2. \tag{1}$$

k and S_n are the transaction and shipping costs, respectively. pQ_n are product expenditures and $\frac{h}{2r}Q_n^2$ is the inventory holding cost. Let $X_n, n \geq 1$ be the random variable corresponding to the elapsed time until the next purchase after the nth visit

$$X_n = \frac{Q_n}{r} = \frac{Q + \epsilon_n}{r}. \tag{2}$$

[7] There are many variations in practice. Netgrocer.com, for example, charges shipping fees that vary not only by order value, but also by region of the United States where the order is shipped.

Lemma 1 *Under free shipping and fixed fees, the long run expected average cost per unit time $LR(Q)$ is*

$$LR(Q) = \frac{k + S + pQ + \frac{h}{2r}(Q^2 + \sigma^2)}{\frac{Q}{r}}.$$

Proof: Because $\epsilon_n, n \geq 1$ are iid, the time until next purchase, $X_n, n \geq 1$, is iid. Defining $N(t)$ as the counting process that specifies the number of visits from time 0 to time t, $\{N(t), t \geq 0\}$ is a renewal process, and the time until the next purchase is therefore a renewal cycle. The total cost incurred at time t is therefore $TC(t) = \sum_{n=1}^{N(t)} \mathrm{C}_n$. Using Ross (1980), the long run average cost per unit time is equal to the expected cost in a renewal cycle $E(C_n)$ divided by the expected length of a renewal cycle $E(X_n)$. Combining equations (1) and (2) we obtain the stated expression for the long run average cost per unit time $LR(Q)$.

The long run average cost per unit time $LR(Q)$ is pseudoconvex in Q since $E(C_n)$ is quadratic in Q and $E(X_n)$ is linear in Q (see Avriel 1976). The optimal policy Q^* is therefore obtained by solving the first-order condition. We now solve for the two base cases: (1) free shipping, and (2) fixed charges.

Free Shipping. When the retailer does not charge for shipping $S_n = 0$ and the solution to the first order condition yields

$$Q_{Free}{}^* = \sqrt{\frac{2kr}{h} + \sigma^2}.$$

This can be simplified to obtain a familiar expression, akin to that for the standard inventory model. To see this let $\hat{K}_{Free} = k + \frac{h}{2r}\sigma^2$ so that

$$Q_{Free}{}^* = \sqrt{\frac{2r\hat{K}_{Free}}{h}}. \tag{3}$$

The optimal long run average cost per unit time is given by $\mathrm{LR}(Q_{Free}{}^*) = L^*_{Free} = \sqrt{2hr\hat{K}_{Free}} + rp = hQ^*_{Free} + pr$.

Fixed Fees. Starting with Lemma 1 and solving the first order condition with $S_n = K$ yields

$$Q_{Fixed}{}^* = \sqrt{\tfrac{2(k+K)r}{h} + \sigma^2} = \sqrt{\tfrac{2r\hat{K}_{Fixed}}{h}} \quad \text{where} \quad \hat{K}_{Fixed} = k + K + \tfrac{h}{2r}\sigma^2. \tag{4}$$

Here $\mathrm{LR}(Q_{Fixed}{}^*) = L^*_{Fixed} = \sqrt{2hr\hat{K}_{Fixed}} + rp = hQ^*_{Fixed} + pr$. Comparing this to the result for free shipping leads to a straightforward and reassuring conclusion. A fixed fee increases the optimal order quantity per site visit, Q^*, and also increases the elapsed time between visits, Q^*/r. Furthermore, the addition of a shipping fee necessarily increases the long run overall cost faced by the shopper.

Value-Contingent Shipping Fees

The cases of free shipping and fixed fees offer straightforward results. Analysis of value-contingent shipping fees provides the more interesting and less obvious insights. The e-tailer now waives the shipping fee when shopper expenditures exceed a certain level, which we denote by T. If expenditures exceed T then $S_n = 0$, and if not $S_n = K$. The total cost incurred on visit n pivots around the free shipping threshold as follows

$$\mathrm{C}_n = \begin{cases} \mathrm{C}_{n,h} = k + pQ_n + \frac{h}{2r}Q_n^2 & \text{if} \quad pQ_n \geq T, \\ \mathrm{C}_{n,l} = k + K + pQ_n + \frac{h}{2r}Q_n^2 & \text{if} \quad pQ_n < T. \end{cases}$$

$C_{n,h}$ ($C_{n,l}$) represents the cost incurred when the shopper's purchase value is higher than (lower than) T. Substituting $Q_n = Q + \epsilon_n$ in the condition we have

$$\mathrm{C}_n = \begin{cases} \mathrm{C}_{n,h} = k + pQ_n + \frac{h}{2r}Q_n^2 & \text{if} \quad \epsilon_n \geq \frac{T}{p} - Q, \\ \mathrm{C}_{n,l} = k + K + pQ_n + \frac{h}{2r}Q_n^2 & \text{if} \quad \epsilon_n < \frac{T}{p} - Q. \end{cases} \tag{5}$$

The expected cost incurred on the n^{th} visit is a weighted sum where the weights are simply the probabilities of exceeding the threshold (and paying no shipping fee), or not. Hence

$$E(\mathrm{C}_n) = \left[1 - \Phi\left(\frac{T}{p} - Q\right)\right] E(\mathrm{C}_{n,h}) + \Phi\left(\frac{T}{p} - Q\right) E(\mathrm{C}_{n,l}). \tag{6}$$

We have the following lemma.

Lemma 2 *Under the value contingent shipping policy, the long run expected average cost per unit time is*

$$LR(Q) = \frac{\left[1 - \Phi(\frac{T}{p} - Q)\right]\left(k + pQ + \frac{h}{2r}(Q^2 + \sigma^2)\right) + \Phi(\frac{T}{p} - Q)\left(k + K + pQ + \frac{h}{2r}(Q^2 + \sigma^2)\right)}{\frac{Q}{r}}. \tag{7}$$

Proof: The argument is the same as in Lemma 1. The total cost incurred at time t is $TC(t) = \sum_{n=1}^{N(t)} \mathrm{C}_n$. Combining equations (5) and (6), we get the expected cost in a renewal cycle $E(C_n)$ and divide it by the expected length of a renewal cycle $E(X_n) = \frac{Q}{r}$ to obtain the stated expression for the long run average cost per unit time $LR(Q)$.

To obtain the probabilities from $\Phi(\cdot)$ we assume that ϵ_n is uniform on an interval $[\alpha, \beta]$. Because the mean of the distribution is zero, ϵ_n is uniform on $[-\beta, \beta],\ \beta > 0$. Behaviorally this says that random elements of website layout and marketing initiatives have both positive and negative effects on quantities purchased, but on average these effects even out. The value of $(T/p - Q)$ combines with the extremes of the uniform distribution to produce one analytical expression and two boundary conditions for the probability that the free shipping threshold is exceeded and the shopper pays no fee.

Specifically, when $(T/p - Q)$ lies within the interval $(-\beta, \beta)$ the probability of paying no fee, $1 - \Phi\left(\frac{T}{p} - Q\right)$, is equal to $\frac{\beta - (\frac{T}{p} - Q)}{2\beta}$. That is, the discrepancy between T/p and Q is not "too large." In the extreme case where the shopper always hits the (low) threshold, $(T/p - Q)$ is *non positive* and less than the lower support of the distribution (i.e., $\frac{T}{p} - Q \leq -\beta$). This means that $1 - \Phi\left(\frac{T}{p} - Q\right)$ is equal to one. Conversely, when the free shipping threshold is impossibly high ($\frac{T}{p} - Q \geq \beta$) the shopper pays the shipping fee with probability one. The collectively exhaustive and mutually exclusive conditions are

$$
(8) \begin{cases}
\Phi(\frac{T}{p} - Q) = \frac{(\frac{T}{p} - Q) + \beta}{2\beta} & \text{and} \quad 1 - \Phi(\frac{T}{p} - Q) = \frac{\beta - (\frac{T}{p} - Q)}{2\beta} & , \text{when } \frac{T}{p} - Q \in (-\beta, \beta], \\
\Phi(\frac{T}{p} - Q) = 0 & \text{and} \quad 1 - \Phi(\frac{T}{p} - Q) = 1 & , \text{when } \frac{T}{p} - Q \leq -\beta, \\
\Phi(\frac{T}{p} - Q) = 1 & \text{and} \quad 1 - \Phi(\frac{T}{p} - Q) = 0 & , \text{when } \frac{T}{p} - Q \geq \beta.
\end{cases}
$$

In deriving the shopping policy implications for different combinations of prices p and free shipping threshold T we will consider all combinations of cost function that arise from the three regions of probability weight. Combining (7) and (8), we obtain a cost function with three branches:

$$
\mathrm{LR}(Q) = \begin{cases}
L_{Fixed}(Q) = \frac{k + K + pQ + \frac{h}{2r}(Q^2 + \sigma^2)}{\frac{Q}{r}} & , \text{when } Q \leq \frac{T}{p} - \beta & \text{(Fixed Branch)} \\
L_{Int}(Q) = \frac{(k + pQ + \frac{h}{2r}(Q^2 + \sigma^2)) + K \frac{(\frac{T}{p} - Q) + \beta}{2\beta}}{\frac{Q}{r}} & , \text{when } Q \in (\frac{T}{p} - \beta, \frac{T}{p} + \beta) & \text{(Intermediate Branch)} \\
L_{Free}(Q) = \frac{k + pQ + \frac{h}{2r}(Q^2 + \sigma^2)}{\frac{Q}{r}} & , \text{when } Q \geq \frac{T}{p} + \beta & \text{(Free Branch)}
\end{cases}
$$

Earlier we showed that the first order conditions determine Q^*_{Fixed} and Q^*_{Free} as the global optimum for $L_{Fixed}(Q)$ and $L_{Free}(Q)$, respectively. Similarly, the global optimum for $L_{Int}(Q)$ obtains through the first order condition as

$$
(9) \qquad Q^* = Q^*_{Int} = \sqrt{{Q^*_{Free}}^2 + \frac{Kr}{\beta h}\left(\frac{T}{p} + \beta\right)}.
$$

Simple algebra allows us to express the value-contingent shipping fee cost function as

$$\text{(10)} \quad \mathrm{LR}(Q) = \begin{cases} L_{Fixed}(Q) & = \frac{h}{2}\left(\frac{{Q^*_{Fixed}}^2}{Q} + Q\right) + pr & \text{, when } Q \leq \frac{T}{p} - \beta \\ L_{Int}(Q) & = \frac{h}{2}\left(\frac{{Q^*_{Int}}^2}{Q} + Q\right) - \frac{Kr}{2\beta} + pr & \text{, when } Q \in (\frac{T}{p} - \beta, \frac{T}{p} + \beta) \\ L_{Free}(Q) & = \frac{h}{2}\left(\frac{{Q^*_{Free}}^2}{Q} + Q\right) + pr & \text{, when } Q \geq \frac{T}{p} + \beta \end{cases}$$

The analysis to determine the optimal policy arising from the cost function (10) is slightly more involved than that following Lemma 1. We proceed in two steps. First, we obtain the optimal shopping policy for each of the three branches of the cost function (10). This leads to five potential optimal order quantities. Second, we rule out one of these and focus on the remaining four to determine the range of $\frac{T}{p}$ over which each is the optimal shopping policy. These steps are detailed in the Appendix.

We assume that $\frac{Kr}{h} < 8\beta^2$. Note that $\frac{h}{2r}$ measures the consumer's unit storage cost over the consumption cycle. We assume this unit storage cost is large enough relative to the shipping fee (i.e., $\frac{h}{2r} > \frac{1}{16\beta^2}K$). In the Appendix we show that, in the presence of contingent shipping, the global optimal order quantity (Q^*) that results from (10) is defined by

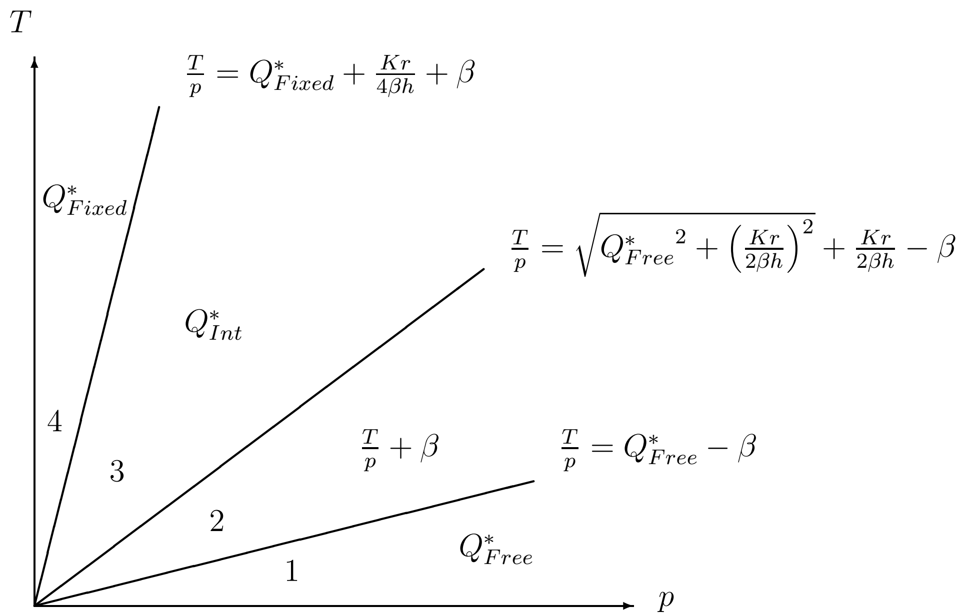
$$\begin{array}{llll} Q^* = Q^*_{Free} & \text{and} \quad \mathrm{LR}(Q^*) = L^*_{Free} = hQ^*_{Free} + pr & \text{if} & \frac{T}{p} < Q^*_{Free} - \beta \\ Q^* = \frac{T}{p} + \beta & \text{and} \quad \mathrm{LR}(Q^*) = L_{Free}(\frac{T}{p} + \beta) & \text{if} & Q^*_{Free} - \beta \leq \frac{T}{p} \leq \sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h} - \beta \\ Q^* = Q^*_{Int} & \text{and} \quad \mathrm{LR}(Q^*) = L^*_{Int} = hQ^*_{Int} - \frac{Kr}{2\beta} + pr & \text{if} & \sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h} - \beta < \frac{T}{p} < Q^*_{Fixed} + \frac{Kr}{4\beta h} + \beta \\ Q^* = Q^*_{Fixed} & \text{and} \quad \mathrm{LR}(Q^*) = L^*_{Fixed} = hQ^*_{Fixed} + pr & \text{if} & \frac{T}{p} \geq Q^*_{Fixed} + \frac{Kr}{4\beta h} + \beta \end{array}$$
(11)

With these expressions in hand we can establish Proposition 1.

Proposition 1 *There are closed form solutions for the optimal order quantities in the presence of contingent shipping.*

The actual optimal order quantity and the associated optimal long run average cost per unit time change depending on the value of T/p. Figure 1 illustrates the optimal solutions with respect to T/p. When the shipping threshold T is very small relative to p (Region 1 of Figure 1), the optimal optimal order quantity is independent of T and equal to Q^*_{Free}, the optimal order quantity under the free shipping regime. As T is very low relative to p, an order quantity of Q^*_{Free} guarantees that on any purchase occasion, the shopper always buys

Figure 1: Optimal Solutions with respect to T/p



enough to satisfy the low dollar purchase requirement for free shipping.[8]

As T increases relative to p and we move beyond Region 1, ordering Q^*_{Free} no longer guarantees free shipping. Two competing forces are now at work. On the one hand the consumer wants to increase the order quantity so as to reach the threshold and avoid the shipping charge K; on the other hand, such an increase in the order quantity leads to undesirably higher inventory costs. As it turns out, the first force completely dominates in Region 2, whereas both forces balance each other in Region 3 (where the optimal order quantity reflects a compromise solution).

In Region 2, the quantity Q^*_{Free} no longer guarantees free shipping on *all* purchase visits as $\frac{T}{p} > Q^*_{Free} - \beta$. The shopper optimally increases her order from Q^*_{Free} to $Q^* = \frac{T}{p} + \beta$, which is the minimum order quantity required in order to reach the free shipping threshold (and avoid paying the fee) on *all shopping occasions*. The consumer increases her order

[8]When $Q^* = Q^*_{Free}$, this guarantees that even for shopping occasions where the actual purchased quantity is at the low end of the support ($\epsilon_n = -\beta$ and $Q_n = Q^* - \beta = \frac{T}{p}$), the free shipping threshold is reached: $pQ_n \geq p(Q^*_{Free} - \beta)$, which is greater than the threshold T in Region 1.

quantity and does what it takes to obtain free shipping, regardless of the associated increase in inventory costs. In Region 2, the incentive to avoid the shipping fee through increased order quantity completely dominates the disincentive associated with higher inventory costs.

As we move into Region 3, T further increases relative to p. The required increase in order quantity to avoid paying the shipping fee becomes substantial, and so does the associated increase in inventory costs. The disincentive associated with higher inventory costs starts to bite and the optimal order quantity is no longer $\frac{T}{p} + \beta$. The shopper strikes a compromise between two opposite and competing incentives — avoiding the shipping fees versus minimizing inventory costs. The optimal balance is now an intermediate quantity $Q^* = Q^*_{Int}$ between Q^*_{Free} and $\frac{T}{p} + \beta$.[9] With $Q^* = Q^*_{Int}$, the consumer will avoid the shipping fee on some purchase visits but not on all of them. She however will carry a lower inventory cost than what she would if she ordered $\frac{T}{p} + \beta$ (in a bid to avoid the shipping fee on all purchase visits).[10] Finally, as we move into Region 4, T becomes too large relative to p. The optimal order quantity is independent of T and equal to Q^*_{Fixed}. As T is too high (relative to p), the shopper can never attain it and always pays K.

With the optimal policy in hand, we can start to understand how p and T in combination influence the rational shopper. We can, for example, explore the implications of a decision to decrease prices while at the same time increasing T. Similarly, we can obtain insight into what should happen when T is lowered (from say \$49 to \$25). Intuitively, lower prices or a lower threshold for free shipping will lead to lower costs for rational shoppers. Considering the two variables together, however, produces more nuanced insights. For a given value of T, higher prices make it more likely that the shopper will exceed T (and pay $S_n = 0$) more often over repeated visits. The net effect therefore requires more detailed elaboration.

[9] In Region 3, we have $Q^* = Q^*_{Int} < \frac{T}{p}$. Also, one can see clearly from equation (9) that $Q^*_{Int} > Q^*_{Free}$.

[10] Our assumption that $\frac{Kr}{h} < 8\beta^2$ insures that the consumer's unit storage cost over the consumption cycle $\frac{h}{2r}$ is large enough relative to the shipping fee K so that the disincentive associated with higher inventory costs has some bite in Region 3. When this condition is not satisfied and the unit storage cost is too small relative to the shipping fee (i.e., $\frac{Kr}{h} \geq 8\beta^2$), then the inventory cost disincentive is too small to ever be a concern, and in that case, even in Region 3, the consumer orders enough to guarantee free shipping on every purchase visit; i.e., $Q^* = \frac{T}{p} + \beta$.

The Relationship Between Free Shipping Thresholds (T) and Prices (p)

When T is either very large (Region 4) or very small (Region 1) relative to p, the optimal long run average cost per unit time is independent of T. We therefore restrict attention to the more interesting situation where *both* the T and p affect the optimal policy (Regions 3 and 2 in Figure 1). In these regions the ratio T/p is of "moderate" size.

In what follows, our focus continues to be on the behavior of the *consumer* expressed through the optimal shopping policy. Any implications for optimal firm behavior are implicit rather than explicit, and we will return to this issue in the concluding section. In order to understand how T and p interact in their effect on consumer behavior, we define an iso-cost curve. The iso-cost curve is a function of T and p such that all points (i.e., every combination of T and p captured by the curve) impose an *identical* level of long run average cost per unit time on the rational shopper. We define the iso-cost curve analytically, illustrate it graphically, and then explore its properties. We begin with Region 3. Here $\frac{T}{p} \in [\sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h} - \beta, Q^*_{Fixed} + \frac{Kr}{4h\beta} + \beta]$ and $Q^* = {Q_{Int}}^*$. From this we obtain the optimal long run average cost per unit time and set that equal to a constant, C

$$L^*_{Int} = hQ^*_{Int} + rp - \frac{Kr}{2\beta} = C.$$

It is now possible to express T as a function of p as follows[11]

$$T_3(p) = p\left(\frac{C-pr}{h} - \beta + \frac{\beta h}{Kr}\left[\left(\frac{C-pr}{h}\right)^2 - {Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2\right]\right). \tag{12}$$

With this expression we analyze the properties of the Region 3 iso-cost curve and summarize them in the following Lemma.

Lemma 3 *The feasible region for the $T_3(p)$ curve is illustrated in Figure 2. The curve is concave between $p = 0$ and $p = r_3$.*

Proof: See Technical Appendix.

[11]The constant C should be greater than the lowest possible cost that can be achieved in this system which is $L^*_{Free} = hQ^*_{Free} + pr$ (i.e., the minimum cost that can be achieved when there are no shipping fees). Thus $\frac{C-pr}{h} \geq Q^*_{Free}$.

[Figures 2 and 3 About Here]

In Region 2, $\frac{T}{p} \in [Q^*_{Free} - \beta, \sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h} - \beta]$. Following the same style of analysis, we set the optimal long run average cost per unit time in Region 2 equal to a constant C (recall that $\frac{C-pr}{h} \geq Q^*_{Free}$) and obtain

$$T_2(p) = p\left(\frac{C-pr}{h} - \beta + \sqrt{\left(\frac{C-pr}{h}\right)^2 - {Q^*_{Free}}^2}\right). \tag{13}$$

Lemma 4 summarizes the properties of the function in Region 2.

Lemma 4 *The feasible region for the $T_2(p)$ curve is illustrated in Figure 3. The curve is concave between $p = 0$ and $p = r_2$.*

Proof: See Technical Appendix.

Thus, we have shown that in Regions 3 and 2 where the ratio T/p is "moderate" such that the threshold and the price both affect the optimal policy, the iso-cost curves defined by $T_3(p)$ and $T_2(p)$ are concave in their relevant regions. Finally, Lemma 5 describes how the curves $T_3(p)$ and $T_2(p)$ intersect at the boundary line $\frac{T}{p} = \sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h} - \beta$ that separates Regions 2 and 3.

Lemma 5 *(a) The curves $T_3(p)$ and $T_2(p)$ cross the boundary of Regions 3 and 2 at the same point p^*, and (b) $T_2(p)$ and $T_3(p)$ are tangent at the point p^* i.e., $T_2'(p^*) = T_3'(p^*)$.*

Proof: See Technical Appendix.

The iso-cost curve $T(p)$ defined by $T_3(p)$ in Region 3 and by $T_2(p)$ in Region 2 is continuous and concave over the Regions 3 and 2. This is shown in Figure 4 and we summarize the result in Proposition 2.

[Figure 4 About Here]

Proposition 2 *In Region 3 and 2, the iso-cost curve $T(p)$ has an inverted-U shape.*

Along the iso-cost curve the rational shopper incurs the same optimal long run average cost per unit time. The first observation is therefore that many combinations of price levels and free shipping thresholds can impose the same total cost, leaving the rational shopper indifferent between such schemes. The inverted-U shape for the iso-cost function also implies the following. Imagine that two different sites set the same value for T. For example, both sites offer free shipping when orders exceed \$25. Holding T constant it is straightforward to see that there are two price points (a low and a high price) that impose the *same* total long run average cost on the rational shopper.[12]

The intuition is as follows. When price levels are relatively low, it is less likely that the value of the order will be large enough to surpass the threshold value T. The shopper is therefore more likely to incur the shipping charge K. So, even though the shopper benefits from lower prices, she seldom gets the benefit of free shipping. Conversely, at the high price point the shopper pays more and is more likely to exceed T and pay no shipping fee. The net effect is that for the *same* value of T two different price points impose the same expected total long run cost on the consumer. This is illustrated in Figure 5.

[**Figure 5 About Here**]

2.4 Model Implications

We explore two issues. First, what do the changes in consumer behavior induced by a change in T suggest a firm should do with prices. Second, whether such changes in T have implications for price dispersion. Proposition 2 provides the basis for the analysis.

Free Shipping Threshold and Retail Price Changes

Suppose a firm lowers its free-shipping threshold. If the goal is to alter behavior but *leave consumers equally well off in terms of their total long run average shopping costs*[13] should

[12] Recall the conditions (Lemmas 3 and 4) under which this is true. It must be the case that the ratio T/p is "moderate" such that both variables influence the optimal policy.

[13] This is a useful goal for the firm. If any threshold change leaves long run average consumer costs constant but changes the buying pattern, consumers should continue to purchase at the same rate. At the same time,

such a change be accompanied by higher or lower prices? To answer this question, consider the impact of an increase in the retail price p on consumers' welfare in terms of their total long run average shopping cost. A price increase has two effects: (1) a direct negative effect, as it increases consumers' purchasing costs, and (2) an indirect positive effect, as it increases the likelihood that consumers' reach the free-shipping threshold over their repeated purchase visits (and hence save on shipping fees over a greater number of purchase visits).

Figure 5 shows that (under the conditions specified in Lemmas 3 and 4) the iso-cost curve is inverted-U shaped. When the current retail price is low and to the left of the peak, the indirect positive effect dominates the direct negative effect. Consumers are better off with the price increase. If the goal of the site is to alter behavior but *leave consumers equally well off in terms of their total long run average shopping costs*, the firm accompanies the price increase with a *raising* of the shipping threshold. This allows the firm to recapture some of the surplus given to consumers who would otherwise benefit from the price increase by triggering the free shipping threshold more often, were it to remain unchanged. Conversely, when the current retail price is already high (to the right of the peak in Figure 5), the direct negative effect dominates. Consumers are worse off with a price increase. If the goal is to again keep consumer costs constant, the firm should accompany the price increase with a *lowering* of the shipping threshold. This compensates consumers for the surplus that would have been taken away had the threshold remained unchanged.

The answer to the initial question is now evident. A firm with already low prices (to the left of the peak in Figure 5) should drop prices further when lowering the threshold. Otherwise total consumer costs are interior to the iso-cost curve and consumers receive a benefit at the firm's expense. For the same reason, a firm with high prices should increase prices further when lowering the free shipping threshold.

Free Shipping Threshold and Internet Retail Price Dispersion

The price-threshold relationship just analyzed can be extended to provide insight into price dispersion. For ease of exposition we compare a high threshold site with a low threshold

profit implications for the firm could be very different. The firm may, for example, prefer to ship four small orders of value $x/4$ to one large order of x.

site.[14] Let LR^*(HT) and LR^*(LT) be the optimal long run average costs per unit time at the high threshold site and the low threshold site, respectively. Imagine both wish to be viewed as equally competitive, in the sense that they impose the *same* total long run average cost on the rational shopper: $LR^*(\text{HT}) = LR^*(\text{LT}) = LR^*$. From Proposition 2 we know that for any given free shipping threshold, there are *two* price points — a low and a high price — that impose the *same* total long run average cost on the rational shopper (see Figure 5). Define $p_1(LT)$ and $p_2(LT)$ to be these prices at the low threshold site, with $p_1(LT) < p_2(LT)$. Similarly, Let $p_1(HT)$ and $p_2(HT)$ be the prices at the high threshold site, with $p_1(HT) < p_2(HT)$. The iso-cost curve in Figure 5 implies that all four price and threshold combinations impose the same long run average cost. That is, LR^* is identical within *and* across sites.

It is easy to see that the price dispersion $p_2(LT) - p_1(LT)$ the low threshold firm can withstand is larger than the price dispersion $p_2(HT) - p_1(HT)$ that the high threshold firm can withstand (see Figure 6).

[**Figure 6 About Here**]

Corollary 1 *Suppose that both Internet sites would like to be viewed as equally competitive in the sense that $LR^*(HT) = LR^*(LT)$. Then the Internet store with the higher free shipping threshold will have a lower price dispersion.*

3 Empirical Analysis

The contribution of this research is to develop a model of consumers' Internet shopping behavior under various shipping fee structures, and derive closed form characterizations of the optimal shopping policies. We show that in the presence of value-contingent shipping, retail price and the free-shipping threshold interact to impact consumers' ordering policy in a complex yet predictable way. This research, therefore, also offers implications that can be examined empirically. We describe our data and two hypotheses to be tested.

[14] The high threshold site (for example) requires purchases of \$49 as qualification for free shipping while the low threshold site requires only \$25.

Data Description

comScore compile data on Internet browsing and purchasing behavior and make it available for academic research. More than 100,000 Internet users were tracked for the period July 1 through December 31, 2002. This panel is a random sample drawn from a cross-section of global Internet users who have given comScore explicit permission to monitor their Internet activity. For each sample member, the data contain information on when and what they purchase, where they purchase from, and how much they pay.

Hypotheses and Findings

We test the following hypotheses:

H_1 (Purchase Quantity and Shipping Thresholds): Average purchase quantities increase with the level of the free shipping threshold.

H_2 (Price Dispersion and Shipping Thresholds): Price dispersion for homogenous goods decreases with the level of the free shipping threshold.

Hypothesis 1 is consisent with data in Lewis et al (2005) and Table 1. To test it further, we examined comScore data for an Internet site with repeat purchasing where the threshold changed. The best candidate site was Amazon.[15] Stated limitations notwithstanding, use of a single site allows us to somewhat control for other exogenous changes in the environment or customer mix that might have also led to changes in the average purchase quantities. From July 1, 2002 to August 24, 2002 orders that exceeded \$49 qualified for free shipping. From August 25, 2002 through October 18, 2002 they qualified at \$25.[16] Under the \$49 regime, the average purchase quantity of products per order was 3.31. Under the \$25 regime it was 2.53. These averages are significantly different ($p < 0.001$). The finding is consistent with H_1. A small sample of within-individual data also supports H_1. There were 45 panelists who

[15]Repeat purchasing takes place at Amazon, however the core produts of books, music, movies and video games depart somewhat from the notion of "consumables" in our analytical model. These products are more likely to be accumulated than "used" in the traditional sense. This limitation aside, these are the best data available through comScore.

[16]This was the only change during the period of the data. Note we are examining two periods of equal length (seven weeks) and have excluded obvious holiday seasons.

qualified for free shipping under *both* regimes. They spent \$17 less per “free-shipping” order when buying under the \$25 threshold and purchased 1.82 fewer items.

The second hypothesis is also tested with comScore data from Amazon.com.[17] comScore assigns products to the following categories: Books, Movies and Video, Music, and Video Games. Using these categories, it is relatively straightforward to obtain exact matches in the log files. Specifically, we must use *exactly the same* product (book, CD, etc.) and compute two observations on price dispersion: (1) price dispersion under the \$49 regime, and (2) price dispersion and under the \$25 regime. This resulted in 40 unique qualifying products and a total of 80 observations for the regression analysis (data are available upon request). We considered every book, CD, DVD and video game that was purchased at least twice during both periods outlined above as a minimum of two observations per item, per period, are needed to compute our measure of price dispersion.

The theory implies that the lower threshold period should have a higher price dispersion for homogenous goods. This follows from Corollary 1 and Figure 6. By considering individual items (books, CDs, DVDs and video games) that are purchased at least twice in each period we have truly “homogenous” goods in that we can compare the change (if any) in price dispersion for a *specific item* across the high (\$49) and low (\$25) threshold regimes. According to Corollary 1 item-specific price dispersion under each regime is simply the difference between the maximum and minimum price for the item during the regime. This definition of dispersion is also consistent with recent work in economics by Sorensen (2001) in his examination of dispersion for prescription drugs. The following regression is used to analyze the relationship between price dispersion PD_i for item i and the threshold

$$PD_i = \alpha_0 + \alpha_1 MUSIC + \alpha_2 BOOK + \alpha_3 MOVIE + \beta LT \tag{14}$$

where $MUSIC$, $BOOK$ and $MOVIE$ are category fixed effects. $LT = 1$ when the free shipping threshold is \$25, and $LT = 0$ when the threshold is \$49. The theory predicts *greater* price dispersion when the threshold value is lower, so we expect $\beta > 0$.

[**Table 2 About Here**]

[17]Proposition 2 and Collorary 1 apply equally to two different sites, or to one site which changes its threshold at different points in time. The only requirement is that one focus on a homogenous good.

Table 2 shows that consistent with H_2, β is significantly greater than zero ($p < 0.05$); price dispersion for homogeneous goods is higher when the threshold value is lower. (Average price dispersion is $10.25 under the $25 regime and $2.39 under the $49 regime.) Moreover, if prices of a given item decline over time one would expect less dispersion (variation) for that item in the second period of the data. Had Amazon set a low threshold ($25) for the first period, and a high threshold ($49) for the second period, evidence for higher variation in the first period could be tainted. More variation under the $25 regime could simply be due to higher price levels at the beginning of the data (rather than attributable to the threshold effect). Fortunately, the naturally occurring ordering of a $49 threshold followed by a $25 threshold period works against this possibility. Thus, we can rule out one potential confound — higher average prices under the low threshold regime.[18]

While the tests of H_1 and H_2 are intended as illustrative rather than exhaustive, they do provide evidence that is directionally consistent with the predictions of the model.

4 Discussion and Conclusion

Free shipping is considered the most effective marketing tactic in e-tailing. Managers affirm this conjecture and recent academic research has shows dramatic effects of shipping thresholds on site traffic and purchase quantities (see Lewis 2005, Lewis et al 2005). Our research complements these insights with an analytical model of how shipping costs influence rational shopping behavior for goods bought repeatedly.

A base scenario where shoppers encounter either fixed fees or free shipping shows that they will increase purchase quantities per site visit and increase inter-visit times when faced with shipping fees. This result is intuitive and borne out by the data. The more complex case of value-contingent shipping offers additional insights. In particular, we show that higher prices are not necessarily bad news for shoppers. While higher prices do increase purchase costs, they also increase the probability that a shopper will hit the free shipping threshold. This is one important way that e-tailing differs from traditional retailing. In offline retailing

[18]Average prices under the $49 and $25 regimes for each category and the p-value for equal means are as follows. Books ($14.69, $16.35, $p < 0.532$), Movies and Video ($21.41, $23.59, $p < 0.134$), Music ($13.24, $12.68, $p < 0.784$) and Video Games ($49.98, $43.74, $p < 0.208$). Taking all observations together, there is also no signifcant difference between average prices in the two periods ($p < 0.411$).

price increases are always negative for shoppers, everything else held constant.

We derive an iso-cost function and show that for any threshold level there are always two prices — a high and a low price — that impose the same long run average cost on a rational shopper. This insight results from the countervailing effects of higher prices just described. They increase purchase costs, but also help the shopper qualify for free shipping. When the site wants to hold consumer long run average costs constant, whether or not a lowering of the threshold should be accompanied by a raising or lowering of the current price was shown to depend on the current price level. A site with an already low price will want to lower prices further to avoid giving up surplus to consumers. A site with already high prices will need to increase them. The result was extended to show that lower thresholds imply greater price dispersion for homogenous goods. Thus our research not only offers insight into how shipping thresholds work, but also into how they interact with the other key marketing mix variable, price. Preliminary empirical analysis of comScore data yields results consistent with key predictions. The empirical results for purchase quantity also concur with those reported in Lewis (2005) and Lewis et al (2005).

Shipping fees will continue to be an important marketing tool for Internet retailers. On March 15, 2005 Amazon CEO Jeff Bezos announced Amazon Prime "all-you-can-eat" express shipping. A fixed annual fee of $79 gives shoppers free two-day shipping and $3.99 overnight shipping (instead of $16.48). Bezos announced "We expect Amazon Prime to be expensive for Amazon.com in the short term. In the long term we hope to earn even more of your business . . ." While we do not consider this variation explicitly, our research is a first attempt at an analytical framework for understanding the effect of such policies. Future work might also consider different classes of shipping service defined by delivery speed.[19] In addition, future analysis could consider firm profitability of different schemes. While we show how thresholds and prices affect consumer behavior through the optimal shopping policy, we do not address the optimal choice of threshold or prices by firms. Whether the firm prefers customers to visit frequently and place small orders or less frequently and place large orders depends upon the cost of shipping and other factors not considered here.

[19]A colleague and heavy user of Amazon expressed distaste for Bezos' Amazon Prime. She compiles an order over time but does not actually execute it until there is sufficient value to hit the threshold (thereby avoiding any shipping fees). Membership in Amazon Prime would cause her to pay $79.

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Appendix

Proof of Proposition 1

Recall the cost function (10):

$$\mathrm{LR}(Q) = \begin{cases} L_{Fixed}(Q) & = & \frac{h}{2}\left(\frac{{Q^*_{Fixed}}^2}{Q} + Q\right) + pr & \text{, when } Q \leq \frac{T}{p} - \beta \\ L_{Int}(Q) & = & \frac{h}{2}\left(\frac{{Q^*_{Int}}^2}{Q} + Q\right) - \frac{Kr}{2\beta} + pr & \text{, when } Q \in (\frac{T}{p} - \beta, \frac{T}{p} + \beta) \\ L_{Free}(Q) & = & \frac{h}{2}\left(\frac{{Q^*_{Free}}^2}{Q} + Q\right) + pr & \text{, when } Q \geq \frac{T}{p} + \beta \end{cases}$$

To determine the optimal shopping policy that arises from the cost function (10), we proceed in two steps. First, we obtained the optimal shopping policy for each of the three branches of the cost function (10). Second, we solve for the global optimum that emerges from this cost function, and determine over which range of $\frac{T}{p}$ each of the candidate value is indeed the global optimal policy.

Step 1

The cost functions $L_{Free}(Q), L_{Fixed}(Q)$ and $L_{Int}(Q)$ in each of the three branches of (10) are pseudoconvex in Q since each one of them is the ratio of a quadratic function by a linear one (Avriel 1976). For each one of these three cost functions, the optimal order quantity on an open interval can therefore be obtained by straightforward differentiation.

Free Branch. In this first extreme situation we have $Q \geq \frac{T}{p} + \beta$ so that $\Phi(\frac{T}{p} - Q) = 0$ meaning that the shopper always buys enough to satisfy the (low) requirement for free shipping. In this case the optimal shopping policy is characterized by

$$Q^* = Q^*_{Free} \text{ if } Q^*_{Free} > \frac{T}{p} + \beta \quad \text{and} \quad Q^* = \frac{T}{p} + \beta \text{ if } Q^*_{Free} \leq \frac{T}{p} + \beta$$

Denote $X_{Free} = Q^*_{Free} - \beta$ and we have:

$$\text{(15)} \qquad \begin{cases} Q^* = Q^*_{Free} \text{ and } \mathrm{LR}(Q^*) = L^*_{Free} = hQ^*_{Free} + pr & \text{if} \quad \frac{T}{p} < X_{Free} \\ Q^* = \frac{T}{p} + \beta \text{ and } \mathrm{LR}(Q^*) = L_{Free}\left(\frac{T}{p} + \beta\right) & \text{if} \quad \frac{T}{p} \geq X_{Free} \end{cases}$$

Fixed Branch. Similarly, in the other extreme situation, it is straightforward to show that the free shipping threshold T is so high that it is never met($Q \leq \frac{T}{p} - \beta$, so that $\Phi(\frac{T}{p} - Q) = 1$), the optimal shopping policy is then characterized by

$$Q^* = Q^*_{Fixed} \text{ if } Q^*_{Fixed} < \frac{T}{p} - \beta \quad \text{and} \quad Q^* = \frac{T}{p} - \beta \text{ if } Q^*_{Fixed} \geq \frac{T}{p} - \beta$$

Denote $X_{Fixed} = Q^*_{Fixed} + \beta$ and we have:

$$\text{(16)} \qquad \begin{cases} Q^* = Q^*_{Fixed} \text{ and } \mathrm{LR}(Q^*) = L^*_{Fixed} = hQ^*_{Fixed} + pr & \text{if} \quad \frac{T}{p} > X_{Fixed} \\ Q^* = \frac{T}{p} - \beta \text{ and } \mathrm{LR}(Q^*) = L^*_{Fixed}\left(\frac{T}{p} - \beta\right) & \text{if} \quad \frac{T}{p} \leq X_{Fixed} \end{cases}$$

Intermediate Branch. In the intermediate situation, we have $Q \in (\frac{T}{p} - \beta, \frac{T}{p} + \beta)$. In this case, the average order quantities and long run average cost per unit time that define the optimal shopping policy are

$$(17) \qquad Q^* = Q^*_{Int} = \sqrt{{Q^*_{Free}}^2 + \frac{Kr}{\beta h}\left(\frac{T}{p} + \beta\right)} \quad \text{and} \quad \mathrm{LR}(Q^*) = L^*_{Int} = hQ^*_{Int} - \frac{Kr}{2\beta} + pr.$$

We have two boundary conditions that result from the condition that $Q^*_{Int} \in (\frac{T}{p} - \beta, \frac{T}{p} + \beta)$:

1. The first condition $Q^*_{Int} < \frac{T}{p} + \beta$ is equivalent to the quadratic $\left(\frac{T}{p} + \beta\right)^2 - 2\frac{Kr}{2\beta h}\left(\frac{T}{p} + \beta\right) - {Q^*_{Free}}^2$ being positive. This quadratic function of $\left(\frac{T}{p} + \beta\right)$ has two roots of opposite signs. The positive root is $\sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h}$. Given that $\left(\frac{T}{p} + \beta\right)$ is non negative, the quadratic is therefore positive if and only if
$$\left(\frac{T}{p} + \beta\right) > \sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h}.$$
2. The second condition $Q^*_{Int} > \frac{T}{p} - \beta$ is always satisfied when $\frac{T}{p} - \beta$ is negative. When it is non negative, the condition is equivalent to the quadratic $\left(\frac{T}{p} - \beta\right)^2 - 2\frac{Kr}{2\beta h}\left(\frac{T}{p} - \beta\right) - {Q^*_{Fixed}}^2$ being negative. This quadratic function of $\left(\frac{T}{p} - \beta\right)$ has two roots of opposite signs. The positive root is $\sqrt{{Q^*_{Fixed}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h}$. Given that $\left(\frac{T}{p} - \beta\right)$ is non negative, the quadratic is therefore negative if and only if
$$\left(\frac{T}{p} - \beta\right) < \sqrt{{Q^*_{Fixed}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h}.$$

Denote

$$X1_{Int} = \sqrt{{Q^*_{Free}}^2 + \left(\tfrac{Kr}{2\beta h}\right)^2} + \tfrac{Kr}{2\beta h} - \beta \quad \text{and} \quad X2_{Int} = \sqrt{{Q^*_{Fixed}}^2 + \left(\tfrac{Kr}{2\beta h}\right)^2} + \tfrac{Kr}{2\beta h} + \beta,$$

then the range of values for $\frac{T}{p}$ over which Q^*_{Int} is a potential global optimum is given by

$$(18) \qquad X1_{Int} < \tfrac{T}{p} < X2_{Int}$$

Summary of Step 1. We clearly have $X_{Free} < X1_{Int}$ and $X_{Free} < X_{Fixed} < X2_{Int}$. Depending on the values of the parameters K, r, h and β we can have either $X_{Fixed} < X1_{Int}$ or the opposite configuration. We summarize our results in the following Figures A1 and A2

Figure A1:
$X_{Fixed} \geq X1_{Int}$

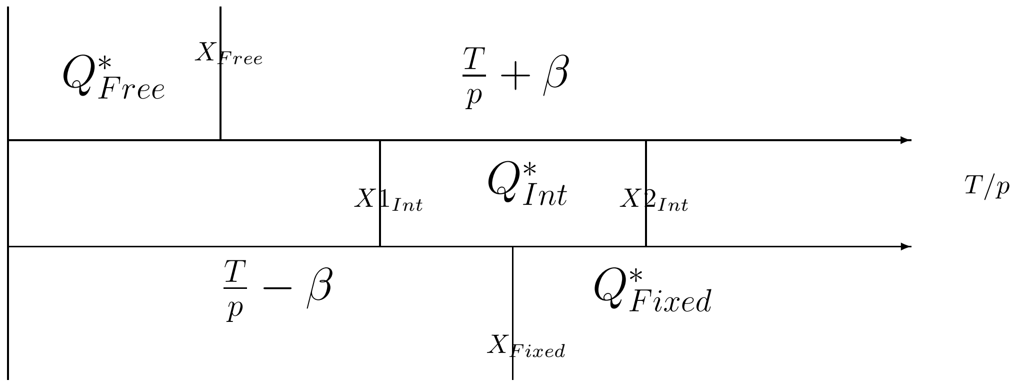
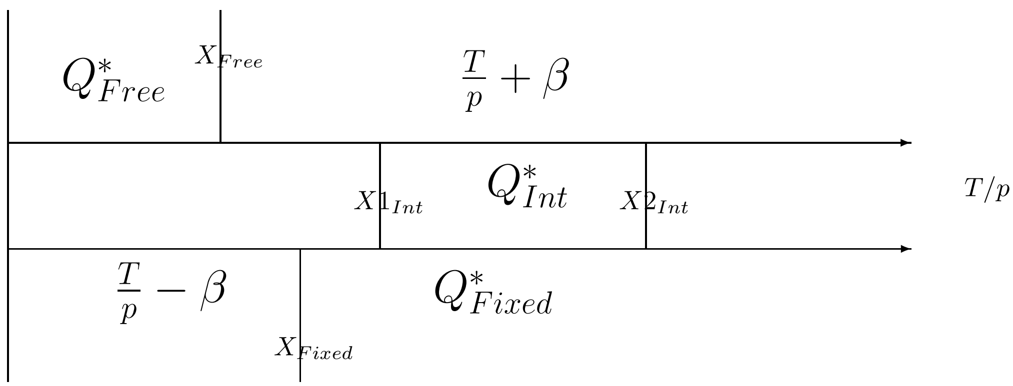


Figure A2:
$X_{Fixed} < X1_{Int}$



Step 2

From Step 1, we have identified five candidate values for the global optimum. We first have a preliminary result where we show that $\left(\frac{T}{p} - \beta\right)$ is never optimal as it is dominated by Q^*_{Free} when $\frac{T}{p} < X_{Free}$ and by $\left(\frac{T}{p} + \beta\right)$ when $\frac{T}{p} \in [X_{Free}, X_{Fixed}]$. To see this, observe that:

first, as Q^*_{Fixed} is the global minimum for L_{Fixed}, we have that $L_{Fixed}\left(\frac{T}{p} - \beta\right) \geq L^*_{Fixed}$; moreover, as expressions (17) and (18) readily indicate, $L^*_{Fixed} \geq L^*_{Free}$, hence $L_{Fixed}\left(\frac{T}{p} - \beta\right) \geq L^*_{Free}$ and thus $\left(\frac{T}{p} - \beta\right)$ is dominated by Q^*_{Free} when $\frac{T}{p} < X_{Free}$;

second, as $\left(\frac{T}{p} + \beta\right) \leq Q^*_{Fixed}$, we have $L_{Fixed}\left(\frac{T}{p} - \beta\right) > L_{Fixed}\left(\frac{T}{p} + \beta\right)$; moreover, as we always have $L_{Fixed}(Q) > L_{Free}(Q)$, it follows that $L_{Fixed}\left(\frac{T}{p} - \beta\right) > L_{Free}\left(\frac{T}{p} + \beta\right)$, and thus $\left(\frac{T}{p} - \beta\right)$ is dominated by $\left(\frac{T}{p} + \beta\right)$ when $\frac{T}{p} \leq X_{Fixed}$, which concludes the argument.

We now proceed to determine the relevant range for $\frac{T}{p}$ over which each of the remaining three candidates is the global optimal shopping policy (Q^*). We do this through a series of intermediary results. Note first, that a consequence of our preliminary result is that (i) Q^*_{Free} is the global optimal when $\frac{T}{p} < X_{Free}$; and (ii) $\left(\frac{T}{p} + \beta\right)$ is the global optimal shopping policy when $\frac{T}{p} \in [X_{Free}, X_{Fixed}]$.

Result 1 *Q^*_{Int} dominates Q^*_{Fixed} if and only if $\frac{T}{p} \leq X_{Fixed} + \frac{Kr}{4\beta h}$*

Proof 1: From expressions (18) and (19), we have that $L^*_{Int} \leq L^*_{Fixed}$ if and only if $hQ^*_{Int} - \frac{Kr}{2\beta} + pr \leq hQ^*_{Fixed} + pr$ if and only if $Q^*_{Int} \leq Q^*_{Fixed} + \frac{Kr}{2\beta h}$. Taking squares on both sides and noting that ${Q^*_{Int}}^2 = {Q^*_{Free}}^2 + \frac{Kr}{\beta h}\left(\frac{T}{p} + \beta\right)$ and ${Q^*_{Fixed}}^2 = {Q^*_{Free}}^2 + \frac{2Kr}{h}$, the condition simplifies to $\frac{T}{p} \leq Q^*_{Fixed} + \frac{Kr}{4\beta h} + \beta = X_{Fixed} + \frac{Kr}{4\beta h}$.

Result 2 *$X1_{Int}$ is smaller than $X_{Fixed} + \frac{Kr}{4\beta h}$ if and only if $\frac{h}{r}$ is greater than $\frac{1}{8\beta^2}K$*

Proof 2: We have $X1_{Int} = \sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} + \frac{Kr}{2\beta h} - \beta$ and $X_{Fixed} = Q^*_{Fixed} + \beta$. Therefore $X1_{Int}$ is smaller than $X_{Fixed} + \frac{Kr}{4\beta h}$ if and only if $\sqrt{{Q^*_{Free}}^2 + \left(\frac{Kr}{2\beta h}\right)^2} \leq Q^*_{Fixed} + 2\beta - \frac{Kr}{4\beta h}$. Taking squares on both sides and noting that ${Q^*_{Fixed}}^2 = {Q^*_{Free}}^2 + \frac{2Kr}{h}$, the condition simplifies to $2Q^*_{Fixed}\left(\frac{Kr}{4\beta h} - 2\beta\right) \leq -\left(\frac{3Kr}{4\beta h} + 2\beta\right)\left(\frac{Kr}{4\beta h} - 2\beta\right)$, which is true if and only if $\left(\frac{Kr}{4\beta h} - 2\beta\right)$ is negative, i.e. $\frac{h}{r}$ greater than $\frac{1}{8\beta^2}K$.

Result 3 *(i) $\left(\frac{T}{p} + \beta\right)$ dominates Q^*_{Fixed} when $X_{Fixed} \leq \frac{T}{p} \leq Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta$, for $\frac{h}{r}$ is smaller than $\frac{1}{2\beta^2}K$; and (ii) Q^*_{Fixed} always dominates $\left(\frac{T}{p} + \beta\right)$ when $\frac{h}{r}$ is larger than $\frac{1}{2\beta^2}K$.*

Proof 3: The condition for $\left(\frac{T}{p} + \beta\right)$ to dominate Q^*_{Fixed} is that $L_{Free}\left(\frac{T}{p} + \beta\right) \leq L^*_{Fixed}$. This is equivalent to the quadratic $\left(\frac{T}{p} + \beta\right)^2 - 2Q^*_{Fixed}\left(\frac{T}{p} + \beta\right) + Q^*_{Free}$ being negative. As ${Q^*_{Fixed}}^2 - {Q^*_{Free}}^2 = \frac{2Kr}{h}$, the condition translates to $\left(\frac{T}{p} + \beta\right) \in [Q^*_{Fixed} - \sqrt{\frac{2Kr}{h}}, Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}}]$. As $Q^*_{Fixed} - \sqrt{\frac{2Kr}{h}} - \beta \leq Q^*_{Fixed} + \beta = X^*_{Fixed}$, we do have that $\left(\frac{T}{p} + \beta\right)$ can dominate Q^*_{Fixed} only when $X_{Fixed} = Q^*_{Fixed} + \beta \leq Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta]$, which can only happen if $\frac{h}{r}$ is smaller than $\frac{1}{2\beta^2}K$; and when the latter condition is satisfied, we have that $\left(\frac{T}{p} + \beta\right)$ dominate Q^*_{Fixed} when $\frac{T}{p} \in [X_{Fixed}, Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta]$.

<u>Optimal solution when consumers' unit storage cost is too small:</u> $\left(\frac{h}{r} \leq \frac{1}{8\beta^2}K\right)$
With Results 1, 2 and 3 we can already describe the optimal solution when consumers' unit storage cost is too small, i.e., $\frac{h}{r}$ is lower than $\frac{1}{8\beta^2}K$. With Result 2 we have that $X_{Fixed} + \frac{Kr}{4\beta h} \leq X1_{Int}$, and therefore we are in the situation described in Figure A2 in Step1. Result 1 rules out Q^*_{Int} as an optimal solution in this situation, and Result 2 determines the relevant range for $\frac{T}{p}$ over which each the remaining two candidates $\left(\frac{T}{p} + \beta\right)$ and Q^*_{Fixed} are optimal. (It is straightforward to check that we always have $Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta \leq X_{Fixed} + \frac{Kr}{4\beta h}$).

We can therefore fully characterize the optimal shopping policy (Q^*) when consumers' unit storage cost is too small: (i.e., $\frac{h}{r}$ is lower than $\frac{1}{8\beta^2}K$) as:

$$Q^* = \begin{cases} Q^*_{Free} & \text{, when} \quad \frac{T}{p} \leq Q^*_{Free} \\ \frac{T}{p} + \beta & \text{, when} \quad Q^*_{Free} \leq \frac{T}{p} \leq Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta \\ Q^*_{Fixed} & \text{, when} \quad \frac{T}{p} \geq Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta \end{cases}$$

<u>Optimal solution when consumers' unit storage cost is large enough:</u> $\left(\frac{h}{r} \geq \frac{1}{8\beta^2}K\right)$
First, observe that Q^*_{Fixed} always dominates $\left(\frac{T}{p} + \beta\right)$ when $\frac{T}{p}$ is greater than $X_{Fixed} + \frac{Kr}{4\beta h}$. This is a direct corollary of Result 3 and from the fact that we always have $Q^*_{Fixed} + \sqrt{\frac{2Kr}{h}} - \beta \leq X_{Fixed} + \frac{Kr}{4\beta h}$). We now prove an additional result:

Result 4 Q^*_{Int} *dominates* $\left(\frac{T}{p}+\beta\right)$

Proof 4: By definition of Q^*_{Int}, we have that $L^*_{Int} = L_{Int}(Q^*_{Int})$ is smaller than $L_{Int}\left(\frac{T}{p}+\beta\right)$. It is easy to check that $L_{Int}\left(\frac{T}{p}+\beta\right) = L_{Free}\left(\frac{T}{p}+\beta\right)$, hence the desired result that L^*_{Int} is smaller than $L_{Free}\left(\frac{T}{p}+\beta\right)$.

We can now describe the optimal solution when consumers' unit storage cost is large enough, i.e., $\frac{h}{r}$ is greater than $\frac{1}{8\beta^2}K$. With Result 2 we have that $X_{Fixed} + \frac{Kr}{4\beta h} \geq X1_{Int}$; Results 1 and 4 yield Q^*_{Int} as the optimal solution for $\frac{T}{p} \in [X1_{Int}, X_{Fixed} + \frac{Kr}{4\beta h}]$; as we observed that Q^*_{Fixed} always dominates $\left(\frac{T}{p}+\beta\right)$ when $\frac{T}{p}$ is greater than $X_{Fixed} + \frac{Kr}{4\beta h}$, then Result 1 yields Q^*_{Fixed} as the optimal solution for $\frac{T}{p}$ greater than $X_{Fixed} + \frac{Kr}{4\beta h} = Q^*_{Fixed} + \frac{Kr}{4\beta h} + \beta$. The optimal shopping policy (Q^*) when consumers' unit storage cost is large enough:(i.e., $\frac{h}{r}$ is greater than $\frac{1}{8\beta^2}K$) can therefore be written as:

$$Q^* = \begin{cases} Q^*_{Free} & \text{, when} \quad \frac{T}{p} \leq Q^*_{Free} \\ \frac{T}{p}+\beta & \text{, when} \quad Q^*_{Free} \leq \frac{T}{p} \leq X1_{Int} \\ Q^*_{Int} & \text{, when} \quad X1_{Int} \leq \frac{T}{p} \leq Q^*_{Fixed} + \frac{Kr}{4\beta h} + \beta \\ Q^*_{Fixed} & \text{, when} \quad \frac{T}{p} \geq Q^*_{Fixed} + \frac{Kr}{4\beta h} + \beta \end{cases}$$

This solution is illustrated graphically in Figure 1 of this article. This completes the proof of Proposition 1.

Table 1: Shopper Behavior at an Online Retailer[a]

	Shipping Fee for Order Size			Order Size
	\$0 to \$50 (Small)	\$50 to \$75 (Medium)	Over \$75 (Large)	Average Value (Std Dev)
Free Shipping	\$0	\$0	\$0	\$46.05 (3.67)
Contingent Free Shipping	\$4.99	\$6.99	\$0	\$64.68 (11.93)

[a] Adapted and reproduced from Lewis, Singh and Fay (2005, Table 2).

Table 2: Coefficients for Price Dispersion Regression[a]

	Coefficient	Std. Err.	t-value	Pr($>\|t\|$)
α_0 (Intercept)	2.655	4.056	0.658	0.515
α_1 (MUSIC)	-3.549	3.410	-1.041	0.301
α_2 (BOOK)	-0.897	3.078	-0.292	0.772
α_3 (MOVIE)	-2.897	3.294	-0.880	0.382
β (**Low Threshold** — \$25)	**3.930**	**1.761**	**2.232**	**0.029**

[a] Adjusted $R^2 = 0.086$, number of observations = 80.

Figure 2: Curve $T_3(p)$

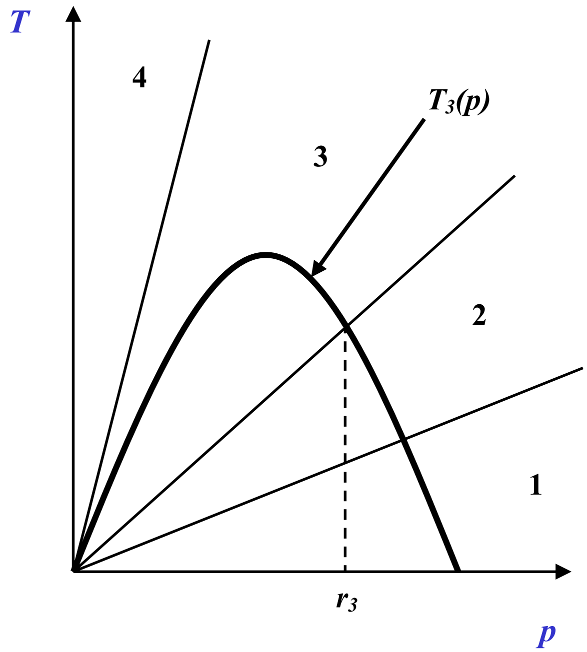


Figure 3: Curve $T_2(p)$

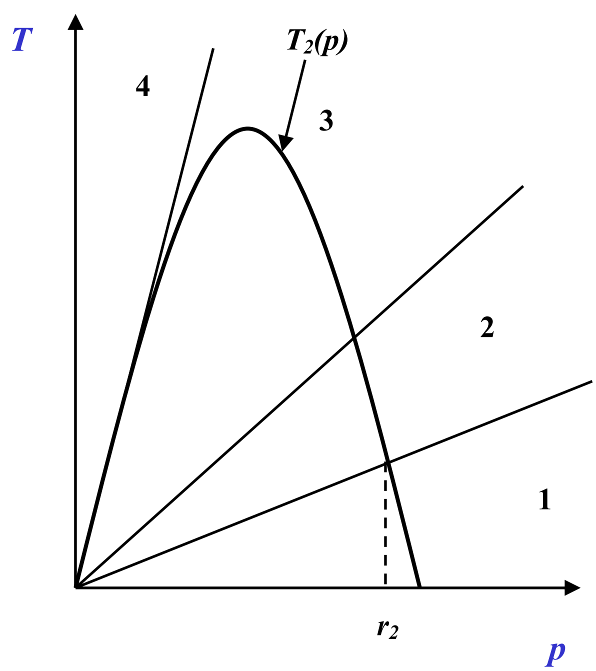


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Figure 4: Iso-Cost Curve *T(p)*

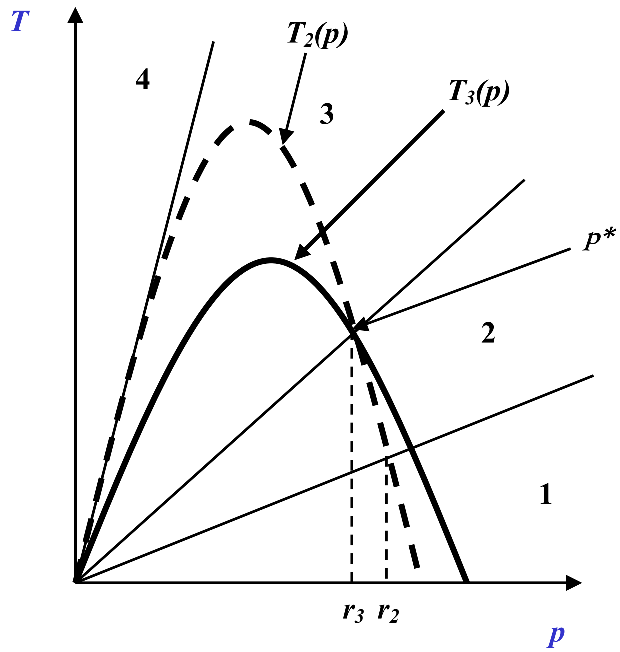

Figure 5: Indifference Points p_1 and p_2

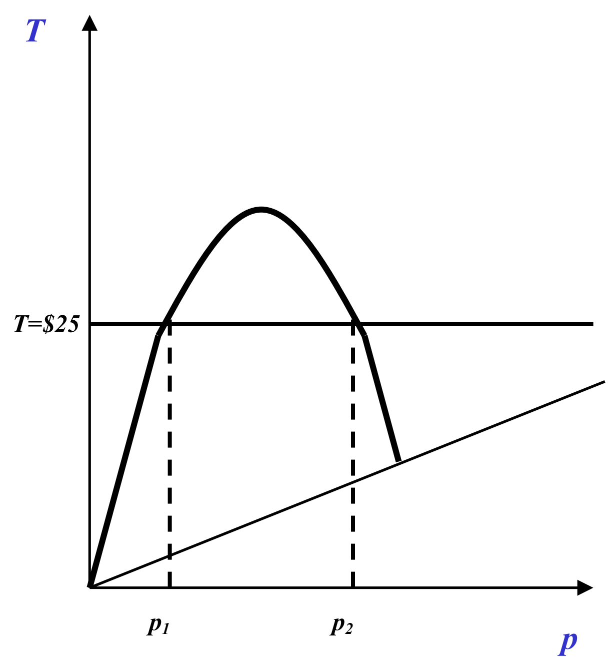

Figure 6: Indifference Points and High and Low Thresholds